Turbulence Closure Modeling in TxBlend

By

The University of Texas at Austin

2000-483-326

Part 2

Turbulence Closure Modeling in TxBLEND

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1 Introduction

During project year 2001, a turbulence closure model was added to the three-dimensional circulation model TxBLEND, developed by the Texas Water Development Board. This turbulence model involves the solution of transport equations for the turbulent kinetic energy and mixing length, as given in [1]. These quantities are then used to compute vertical turbulent mixing coefficients. Preliminary numerical testing of the modified code has been performed for data from Corpus Christi Bay, provided by Dr. Junji Matsumoto.

Below we outline the mathematical equations describing the turbulence model, discuss briefly its implementation within TxBLEND, and present some numerical results.

2 Mathematical Model

The turbulence closure model follows the work of Mellor and Yamada, Galperin *et al* [2] and Blumberg *et al* [3].

Defining

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + v \cdot \nabla,$$

where v is the three-dimensional velocity vector and

$$\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}),$$

the model consists of equations for the the turbulent kinetic energy q^2 and mixing length l:

$$\frac{dq^2}{dt} - \frac{\partial}{\partial z}(N_q \frac{dq^2}{dz}) = 2[N_m((\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial z})^2) + \frac{g}{\rho_0} N_h \frac{\partial \rho}{\partial z}] - 2[\frac{q^3}{B_1 l}], \tag{1}$$

$$\frac{dq^2 l}{dt} - \frac{\partial}{\partial z}(N_q \frac{dq^2 l}{dz}) = lE_1[N_m((\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial z})^2) + \frac{g}{\rho_0} N_h \frac{\partial \rho}{\partial z}] - lW[\frac{q^3}{B_1 l}]. \tag{2}$$

The verticular turbulent mixing coefficients are given by

$$N_m = qls_m, \tag{3}$$
$$N_h = qls_h, \tag{4}$$
$$N_q = qls_q, \tag{5}$$

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I	II	III
$A_1 = .92$	$g_0 = 1 - 6A_1/B_1$	$g_0 = .66747$
$A_2 = .74$	$g_1 = 6A_1 + B_2$	$g_1 = 15.620$
$B_1 = 16.6$	$g_2 = A_1(g_0 - 3C_1)$	$g_2 = .39327$
$B_2 = 10.1$	$g_3 = 3A_1A_2[(B_2 - 3A_2)g_0 - 3C_1g_1]$	$g_3 = 3.0858$
$C_1 = .08$	$g_4 = 3A_2g_1$	$g_4 = 34.676$
$E_1 = 1.8$	$g_5 = 9A_1A_2$	$g_5 = 6.1272$
$E_2 = 1.33$	$g_6 = A_2g_0$	$g_6 = .49393$
$E_3 = .25$		
$\kappa = .4$		

Table 1: Summary of constants used in the vertical turbulence closure model. Column I contains the Mellor-Yamada [1] experimental constants, and E_3 from Blumberg *et al* [3] and von Karman's constant κ. Column II contains the formulas from Galerpin *et al* [2] used in determining s_m and s_h. Column III gives their actual numerical values.

where s_m, s_h are algebraic functions of the local stratification $G_h = \frac{l^2}{q^2}\frac{g}{\rho_0}\frac{\partial\rho}{\partial z}$ and s_q is a constant. N_m is the vertical diffusion coefficient used in the momentum equations, N_h is used in the temperature and salinity transport equations, and N_q is the vertical diffusion coefficient in the equations for q^2 and q^2l.

In these equations

$$s_m = \frac{g_2 - g_3G_h}{(1 - g_4G_h)(1 - g_5G_h)}, \tag{6}$$

$$s_h = \frac{g_6}{1 - g_4G_h}, \tag{7}$$

$$s_q = 0.2, \tag{8}$$

where g_1-g_6 are given in Table I.

As in Galerpin *et al*, an upper bound on the mixing length is enforced:

$$l \leq \frac{.53q}{\sqrt{-\frac{g}{\rho_0}\frac{\partial\rho}{\partial z}}} \tag{9}$$

and G_h is modified if necessary so that $G_h \leq 0.0233$. This prevents negative diffusivities from being computed in (3)-(5).

The boundary conditions on the bottom of the domain are Dirichlet conditions

$$q^2 = B_1^{2/3}u_*^2, \tag{10}$$

where $u_*^2 = C_d|v_b|^2$, where C_d is the bottom stress drag coefficient and v_b the bottom velocity, and

$$l = \kappa\xi_b. \tag{11}$$

At the free surface, no-flux conditions are prescribed on q^2 and l.

The wall proximity function W in (2) is taken from Blumberg *et al* [3]:

$$W = 1 + E_2 \left[\frac{l}{\kappa(z - z_b + \xi_b)} \right]^2 + E_3 \left[\frac{l}{\kappa(\zeta - z + \xi_s)} \right]^2 , \qquad (12)$$

where ξ_b and ξ_s have been set equal to 1 (ft).

Initial conditions must also be prescribed for q^2 and l. These have been taken to be constant. Also, minimum values of N_m, N_h and N_q should be chosen.

3 Modifications to the code

Only a few modifications/additions to the code were required. The same discretization strategy used in TxBLEND for temperature and salinity transport were mimicked for the solution of q^2 and q^2l. An additional variable 'MYCLOSURE' has been added to the code to indicate the type of closure model. If 'MYCLOSURE=CONSTANT,' then the turbulence closure model is skipped and constant values are assigned to N_m and N_h as determined in the subroutine **NzKz3**. If 'MYCLOSURE=25,' then the turbulence model is used. MYCLOSURE is hardwired in **MAIN**.

The turbulence variables and minimum values are initialized in a new subroutine, **initTurb**. The coefficients N_m, N_h and N_q are computed in the new subroutine **Galperin**. Subroutines **COEFQ2** and **COEFQ2L** were also added for the implicit solution of q^2 and q^2l in the vertical direction.

Existing subroutines which were modified include **MAIN** and **COEFGEN10**. All changes to existing code are clearly marked and delineated by comment cards starting with

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c**cnd.
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4 Results

A 30 day simulation was performed using data from Corpus Christ Bay obtained from Junji Matsumoto. The finite element mesh contains 6786 nodes and 11992 elements, with up to 6 layers in the vertical direction. The start date of the simulation was May 1, 1987. A time step of 100 seconds was chosen, which was about the largest time step allowed without one or more solution variables blowing up after a few steps. Initial values of q^2 and q^2l were both chosen to be .001. Minimum values of N_m, N_h and N_q were set to .001, .0005 and .001 ft^2/s, respectively.

The preliminary results obtained with the new turbulence model indicate a larger variation in the vertical direction, especially in the velocity solution, than in the previous version of TxBLEND where the vertical diffusion coefficient was assumed constant with z, at least in this 30 day window. Variations of salinity with depth were less pronounced. Contour plots of salinity at days 17.4, 19.7, 22.6, 25.5 and 28.9 days in the top layer and in layer 3 are given in Figures 1-10.

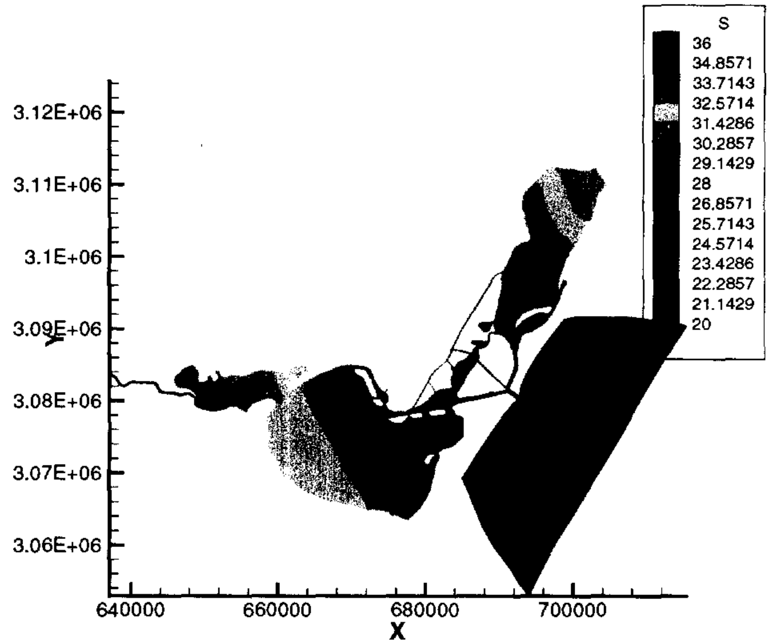

Figure 1: Salinity profile, day 17.4, top layer

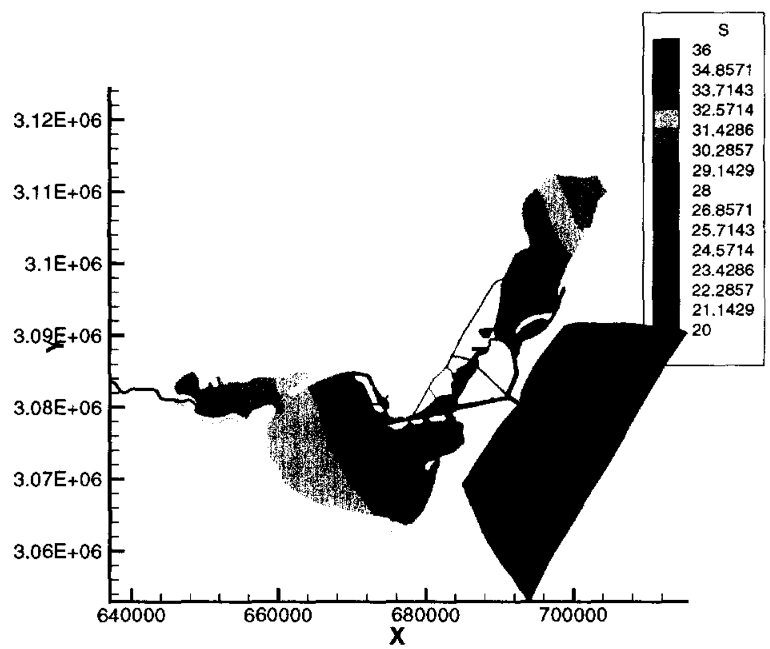

Figure 2: Salinity profile, day 17.4, layer 3

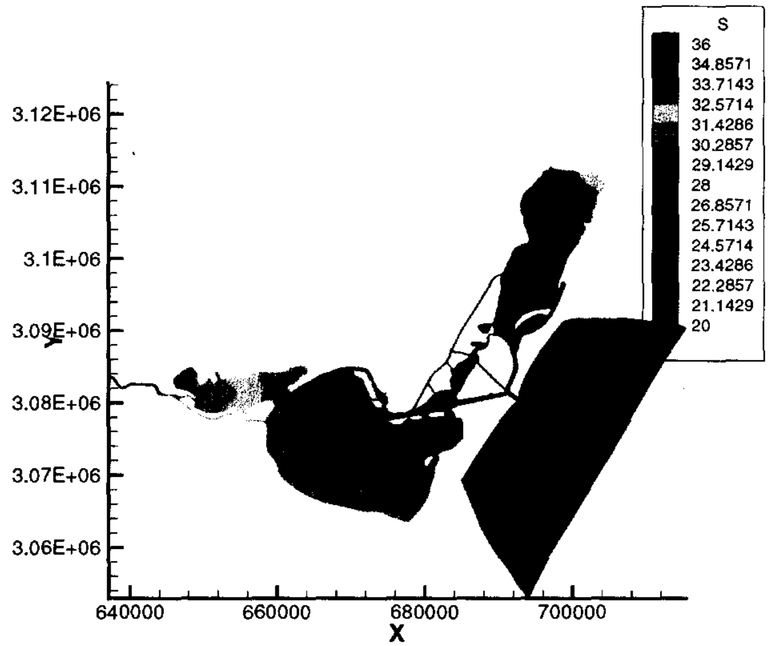

Figure 3: Salinity profile, day 19.7, top layer

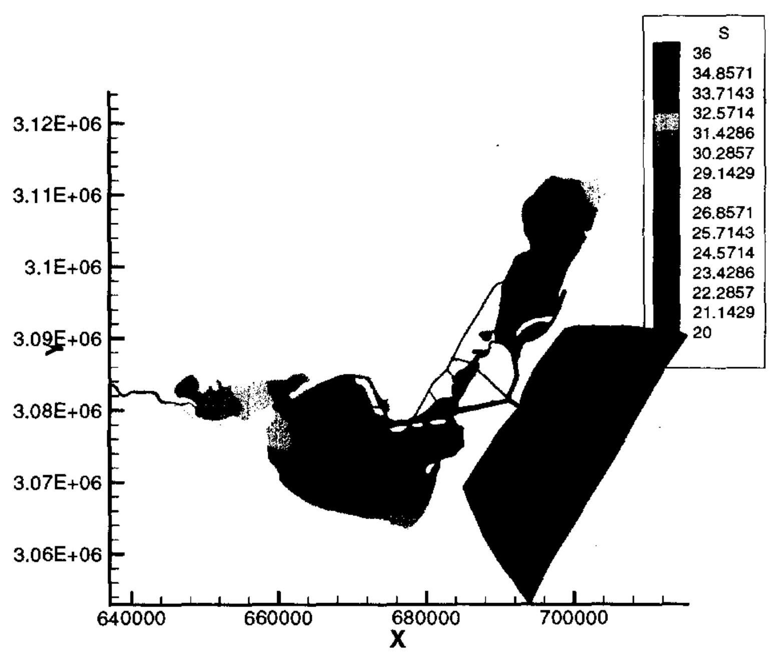

Figure 4: Salinity profile, day 19.7, layer 3

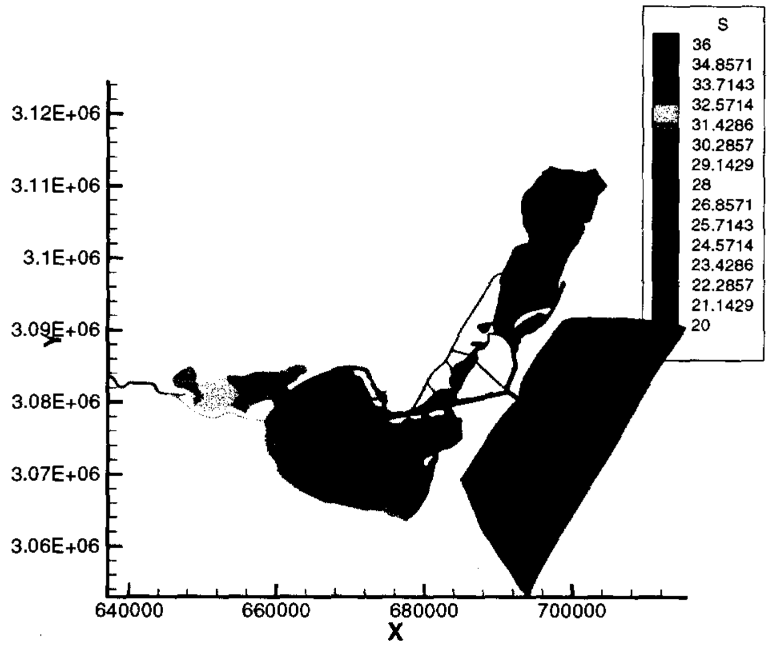

Figure 5: Salinity profile, day 22.6, top layer

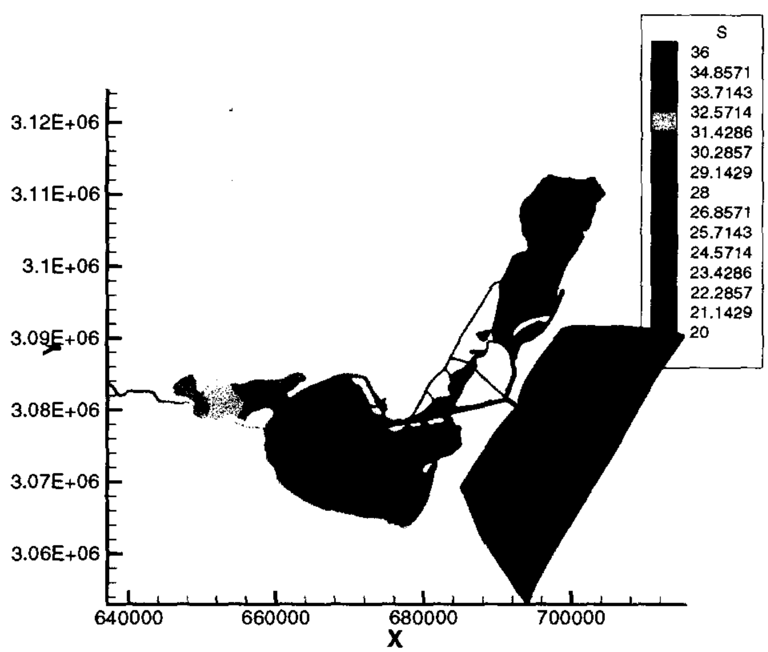

Figure 6: Salinity profile, day 22.6, layer 3

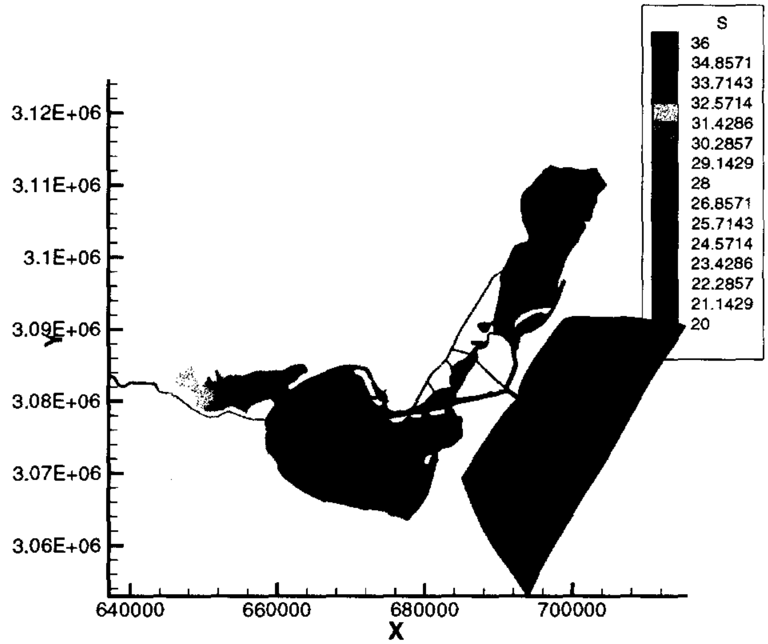

Figure 7: Salinity profile, day 25.5, top layer

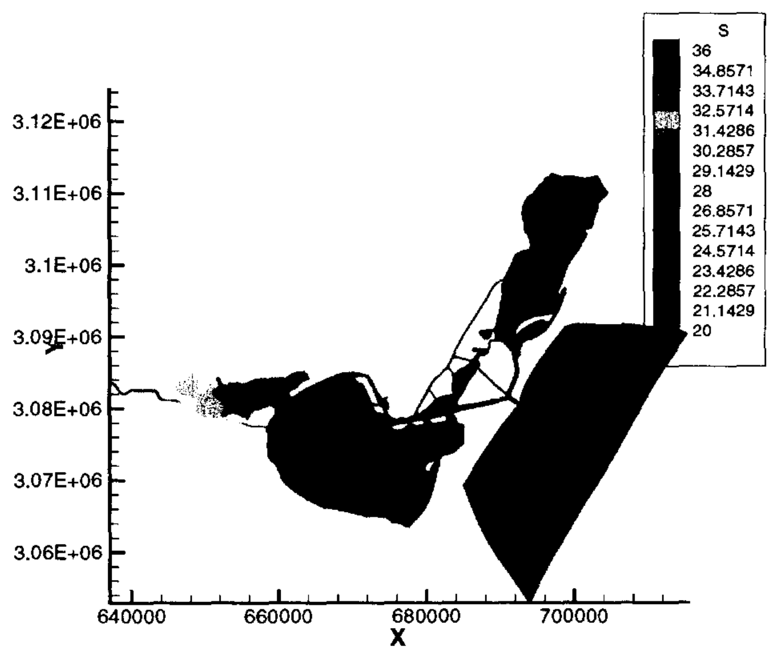

Figure 8: Salinity profile, day 25.5, layer 3

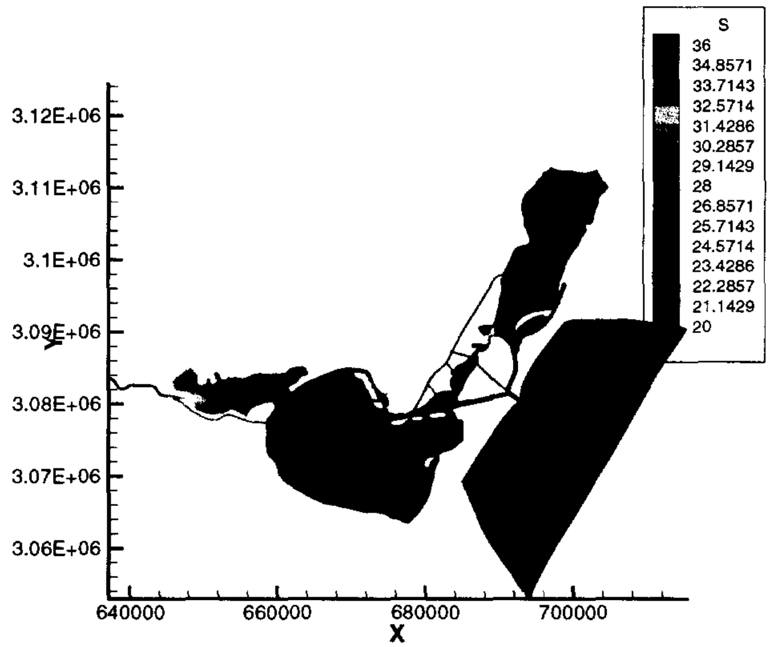

Figure 9: Salinity profile, day 28.9, top layer

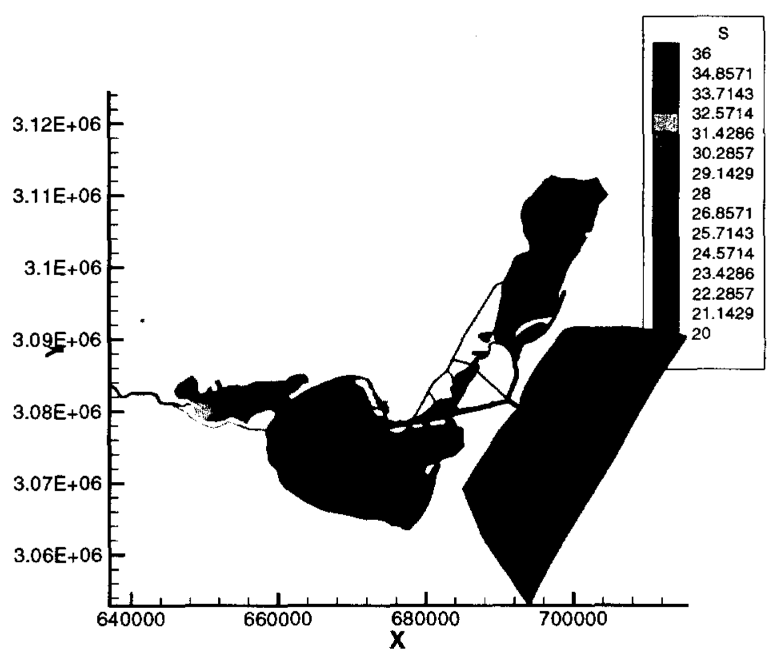

Figure 10: Salinity profile, day 28.9, layer 3

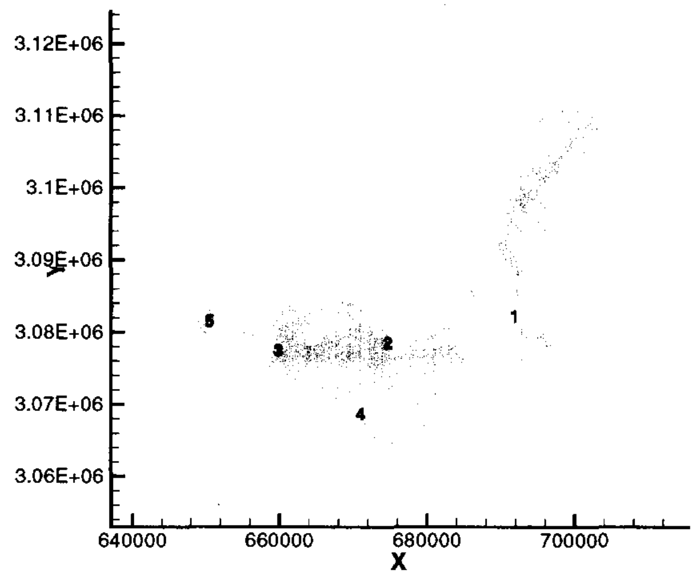

Figure 11: Locations 1-5 indicate where velocities and salinity are plotted below

Next, we show vertical profiles of velocity and salinity at 5 locations within the domain: (1) near the entrance to the ship channel, (2) midway through the ship channel, (3) near the harbor bridge, (4) the southern part of Corpus Christi Bay, and the (5) western part of Nueces Bay. These locations are shown on the finite element mesh in Figure 11. Profiles at 25 days through 26 days were plotted at 3-hour intervals. Each plot consists of the magnitude of the velocity, multiplied by 1 or -1 depending on the direction (1 if direction is SW or NW, -1 otherwise). Velocities are in feet/sec. Figure 12 is for the location near the entrance to the ship channel, Figure 13 for the midway point of the ship channel, Figure 14 for the harbor bridge, Figure 15 for the southern part of Corpus Christi Bay, and Figure 16 for the western part of Nueces Bay.

Vertical profiles of salinity at each of the 5 locations above are given in Figures 17-21. In some locations, in particular location 1, the salinity is constrained to its maximum allowable value of 38 ppt.

5 Conclusions

A turbulence model has been added to the TxBLEND code developed by the TWDB. Preliminary testing on Corpus Christi Bay data has been performed. Further testing and verification of the model is needed.

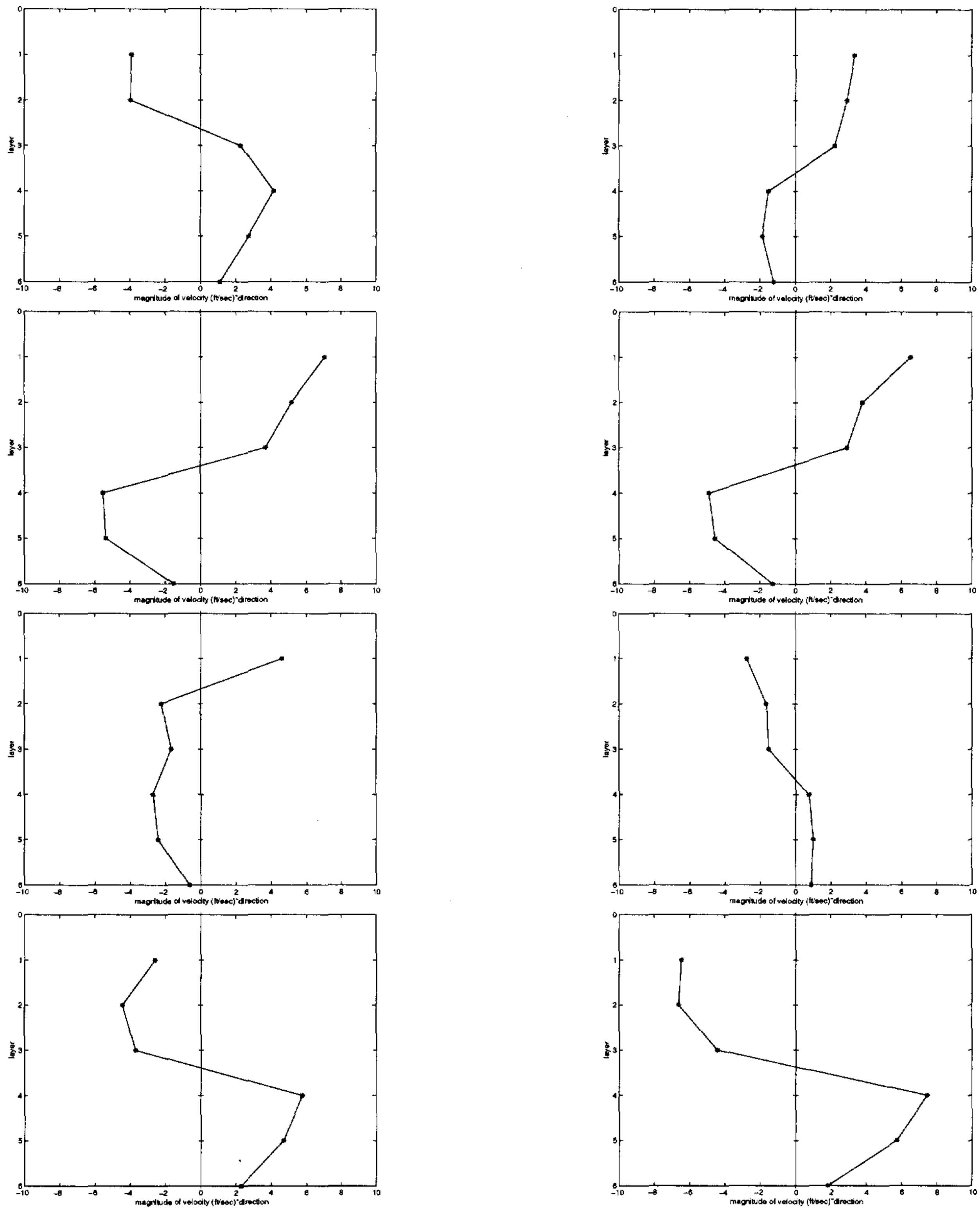

Figure 12: Vertical profiles of velocities at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at entrance to ship channel (location 1)

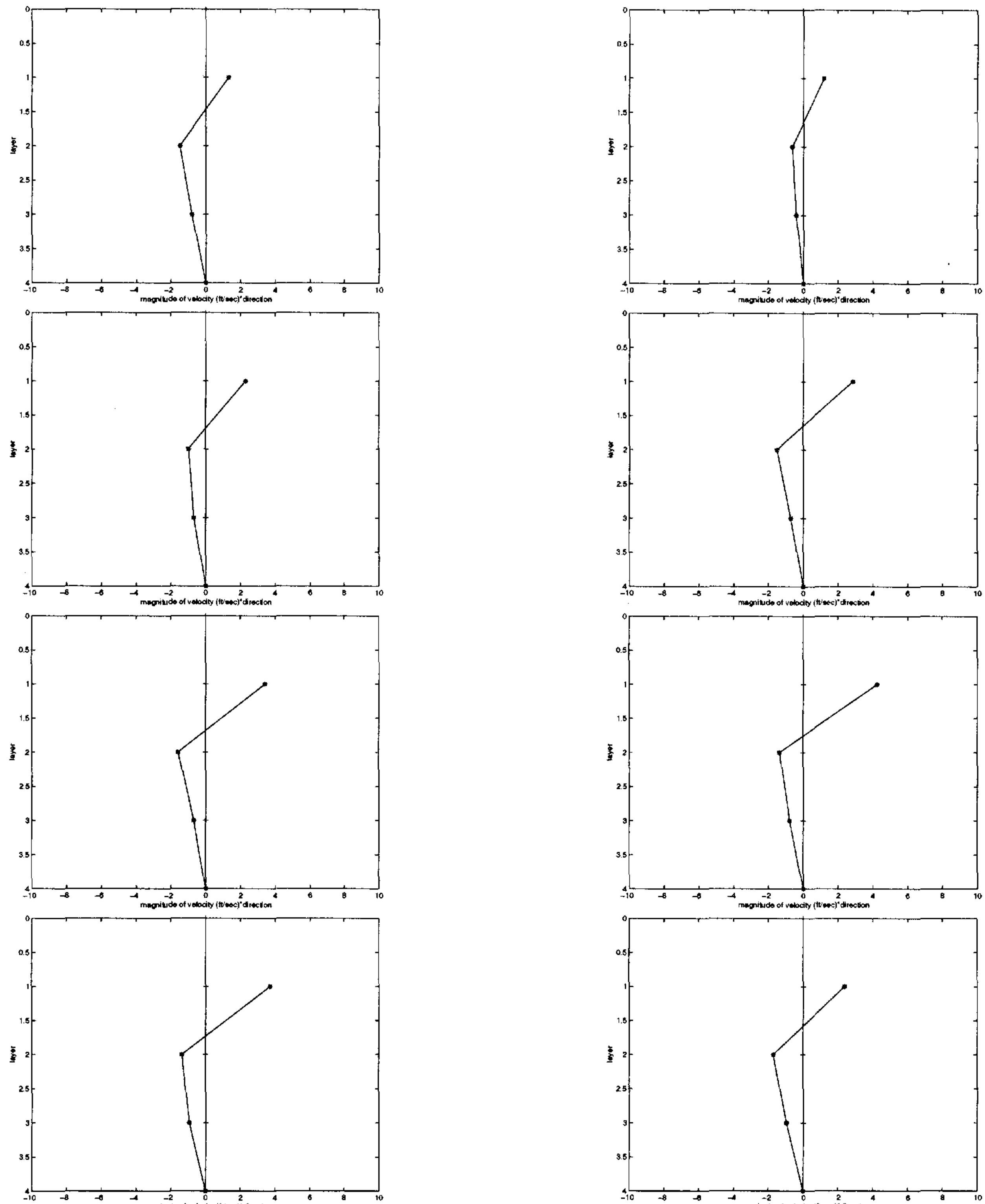

Figure 13: Vertical profiles of velocities at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at mid ship channel (location 2)

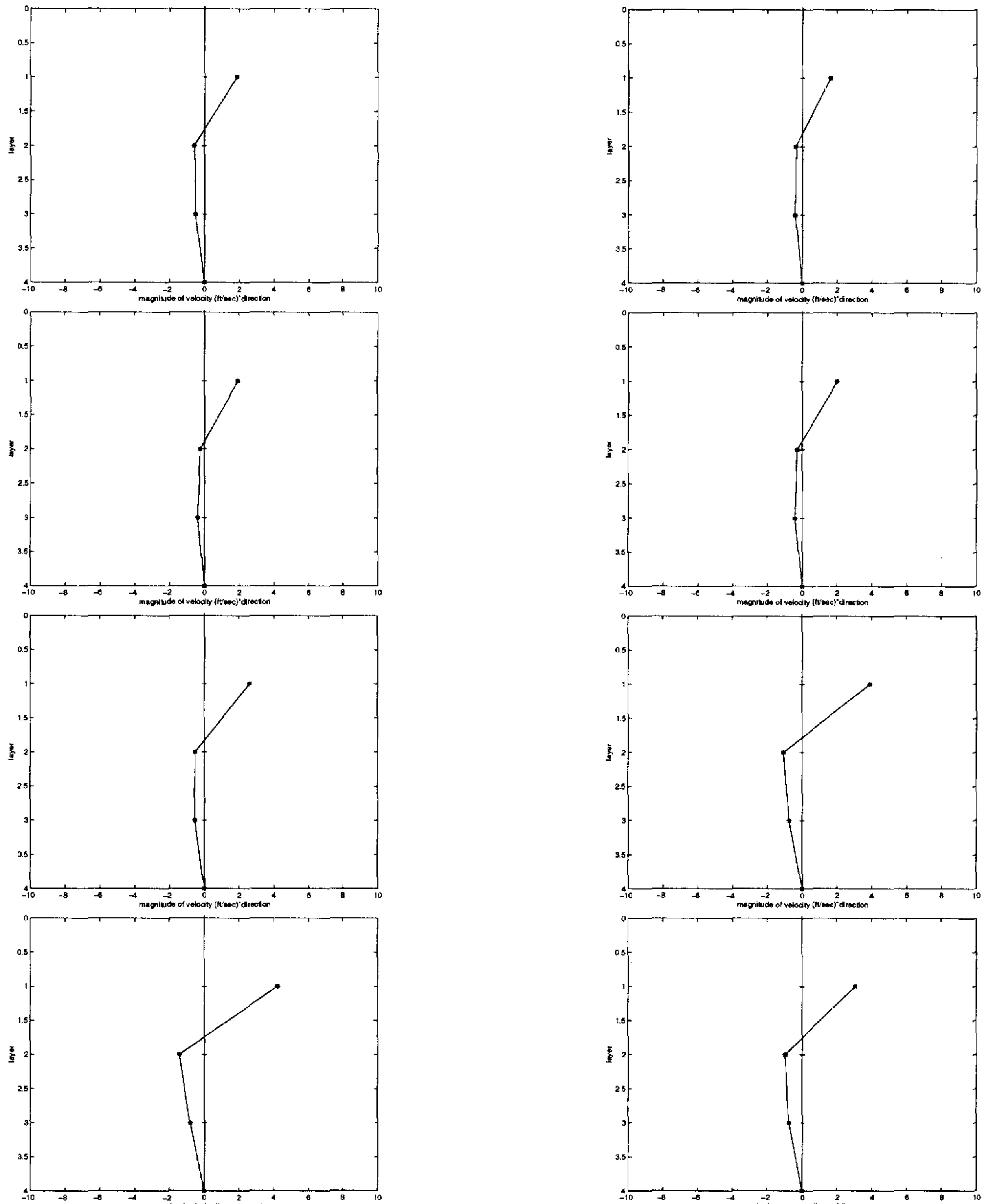

Figure 14: Vertical profiles of velocities at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at harbor bridge (location 3)

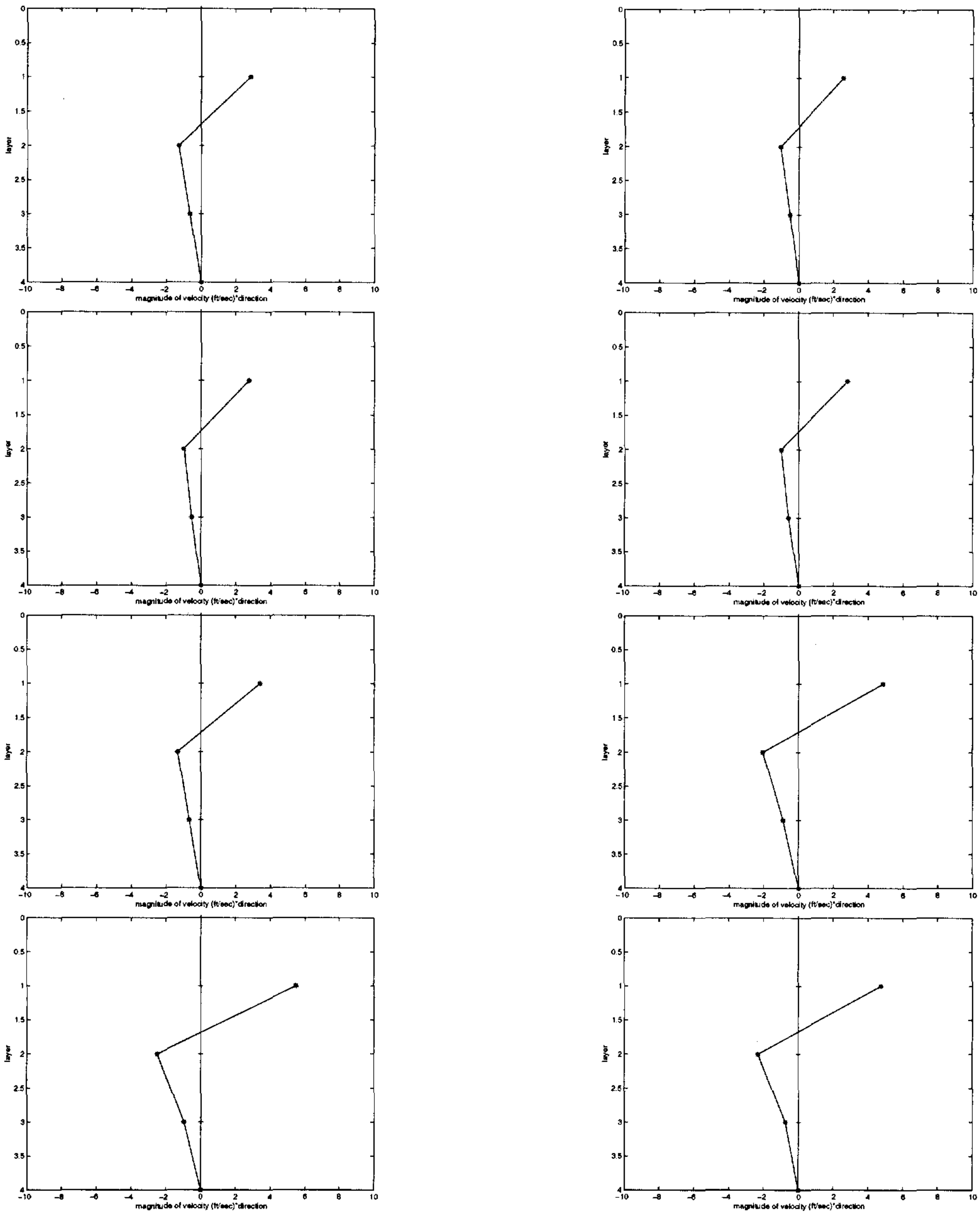

Figure 15: Vertical profiles of velocities at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at Corpus Christi Bay (location 4)

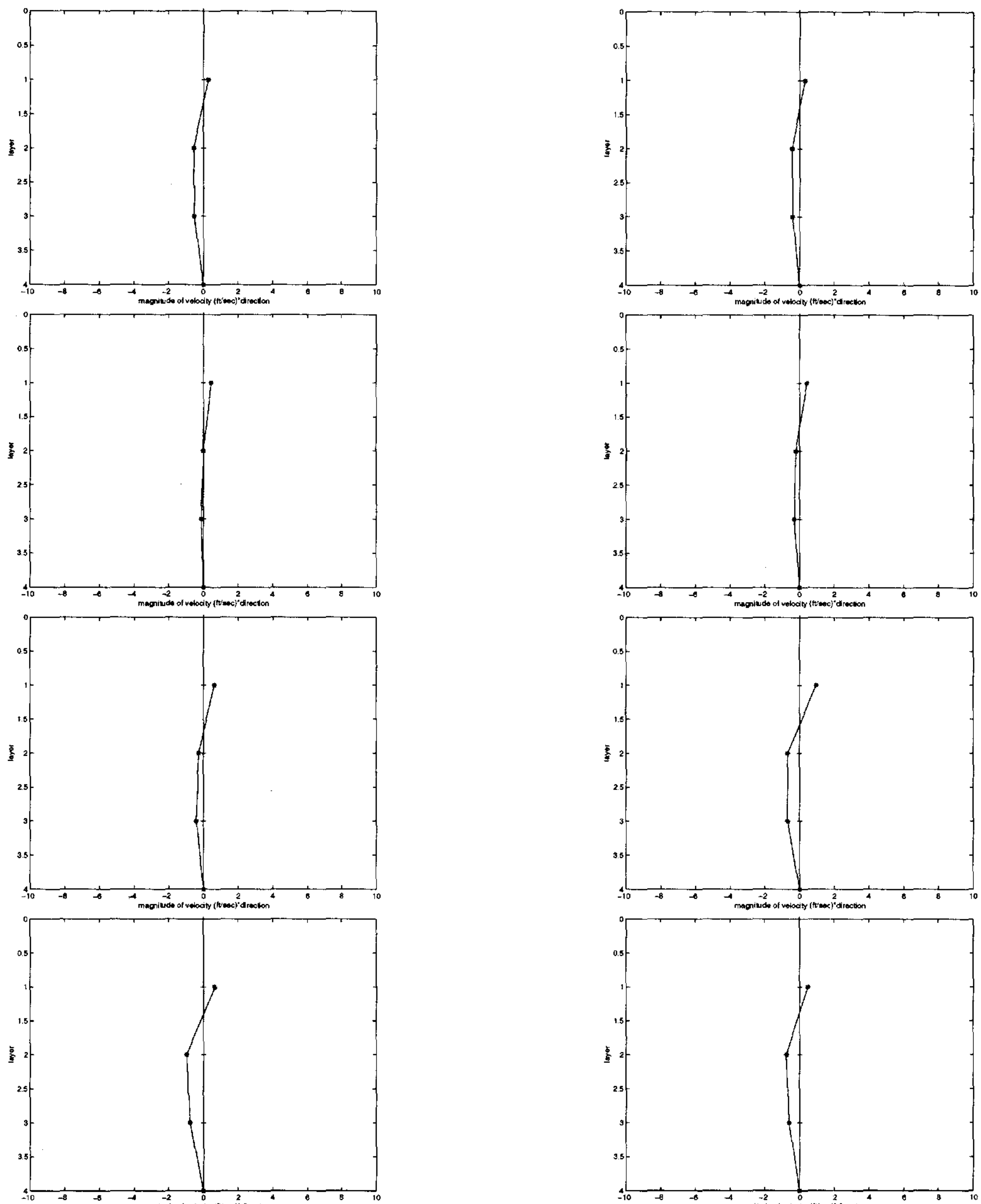

Figure 16: Vertical profiles of velocities at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at Nueces Bay (location 5)

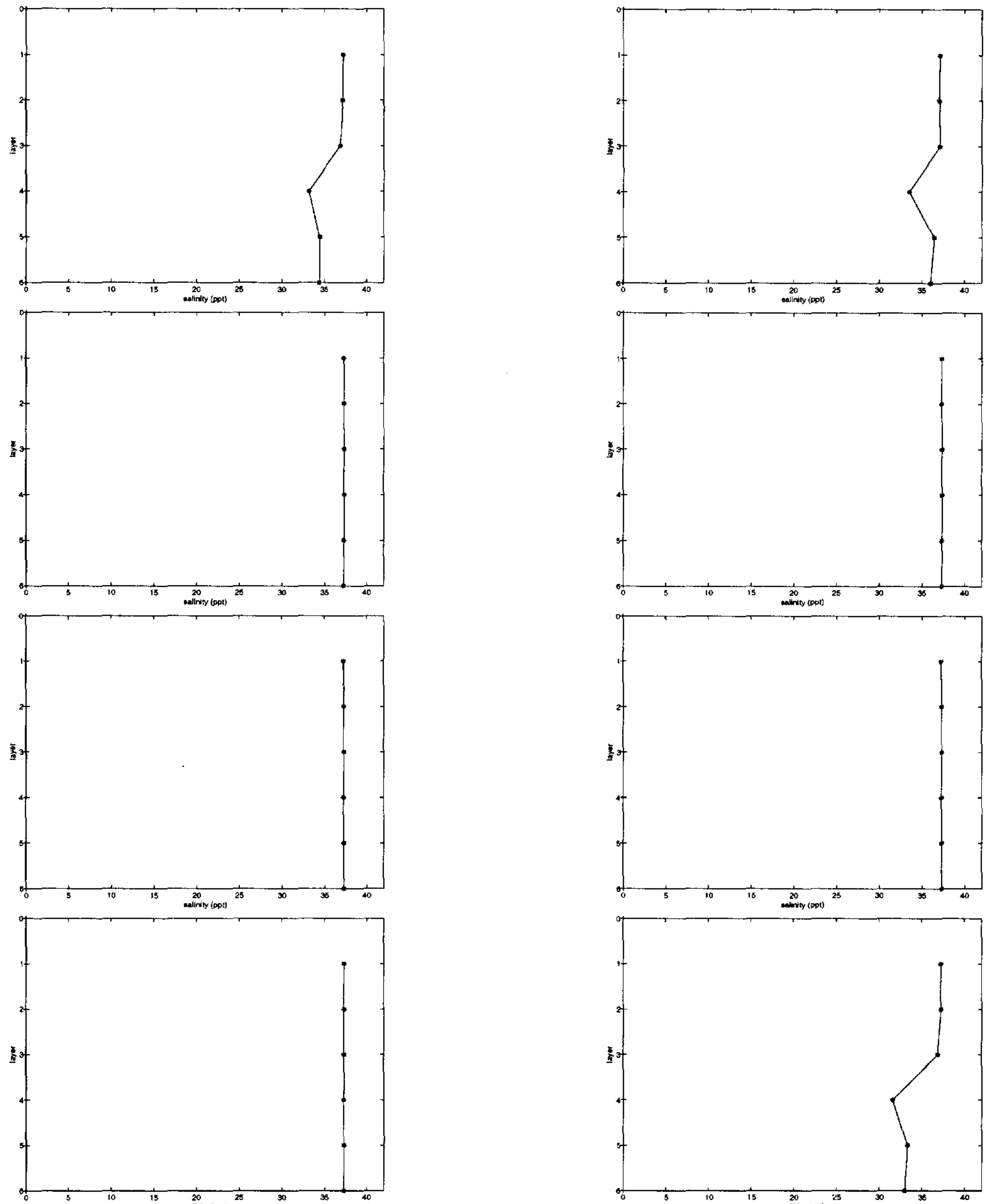

Figure 17: Vertical profiles of salinity at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at entrance to ship channel (location 1)

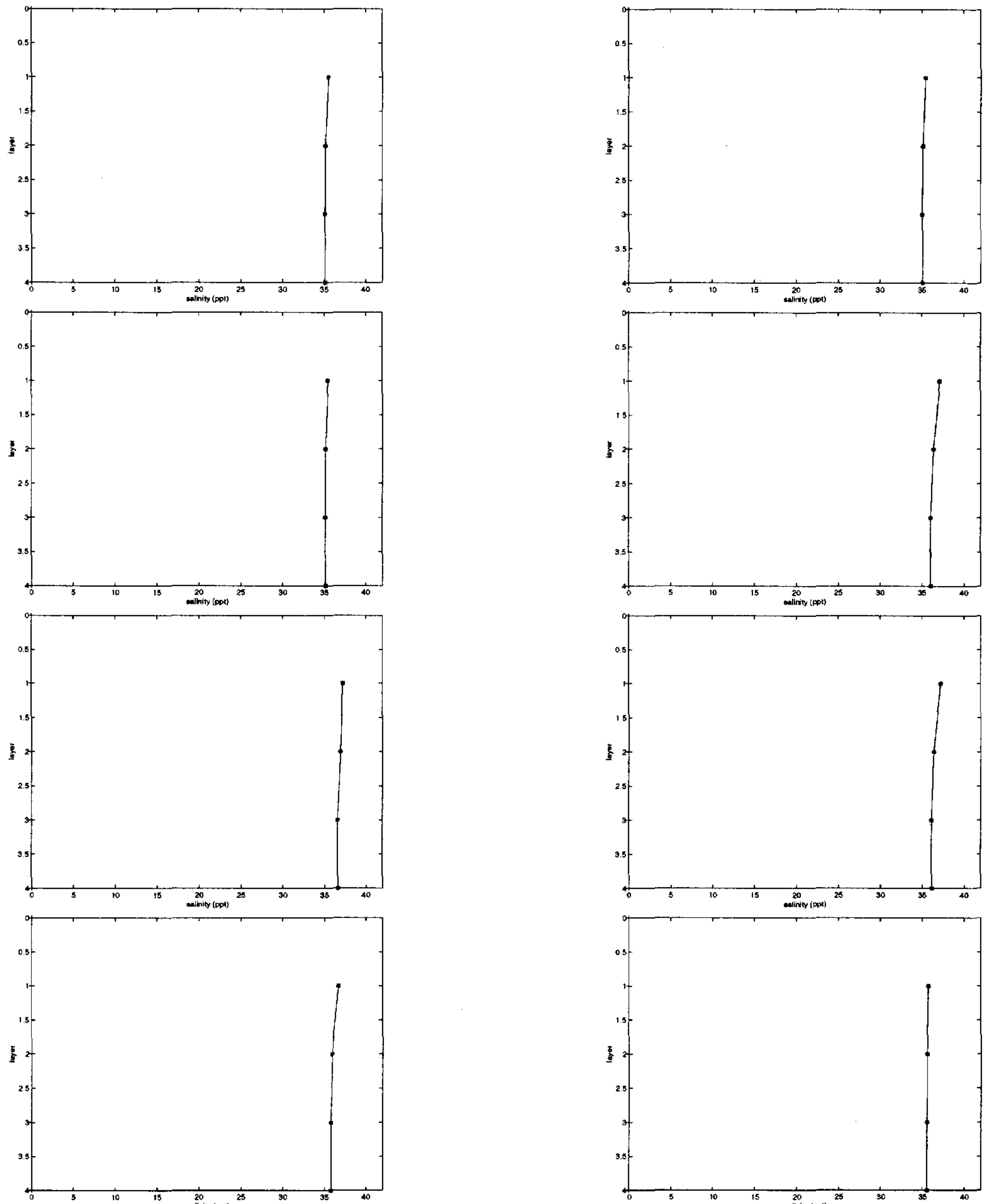

Figure 18: Vertical profiles of salinity at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at mid ship channel (location 2)

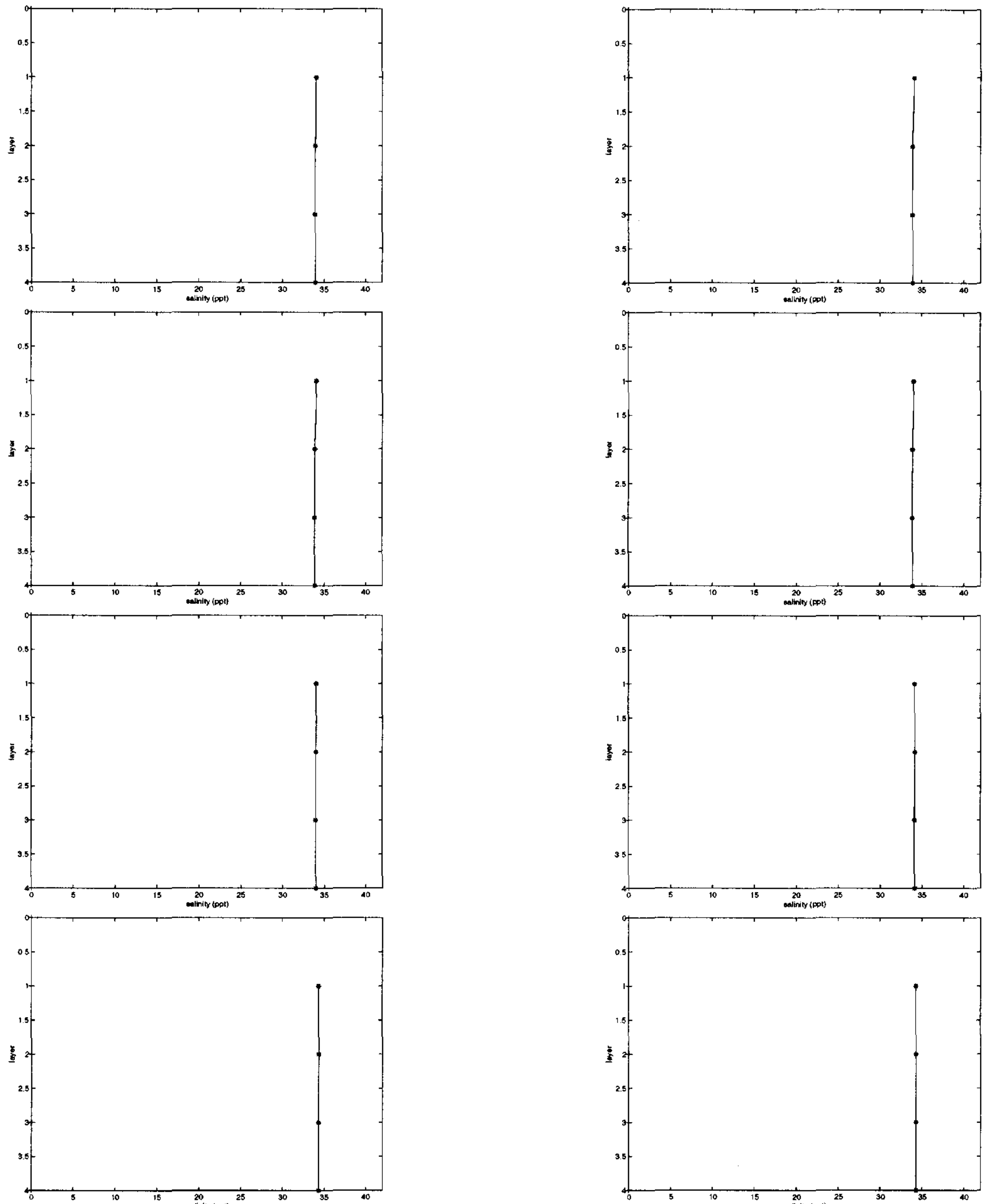

Figure 19: Vertical profiles of salinity at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at harbor bridge (location 3)

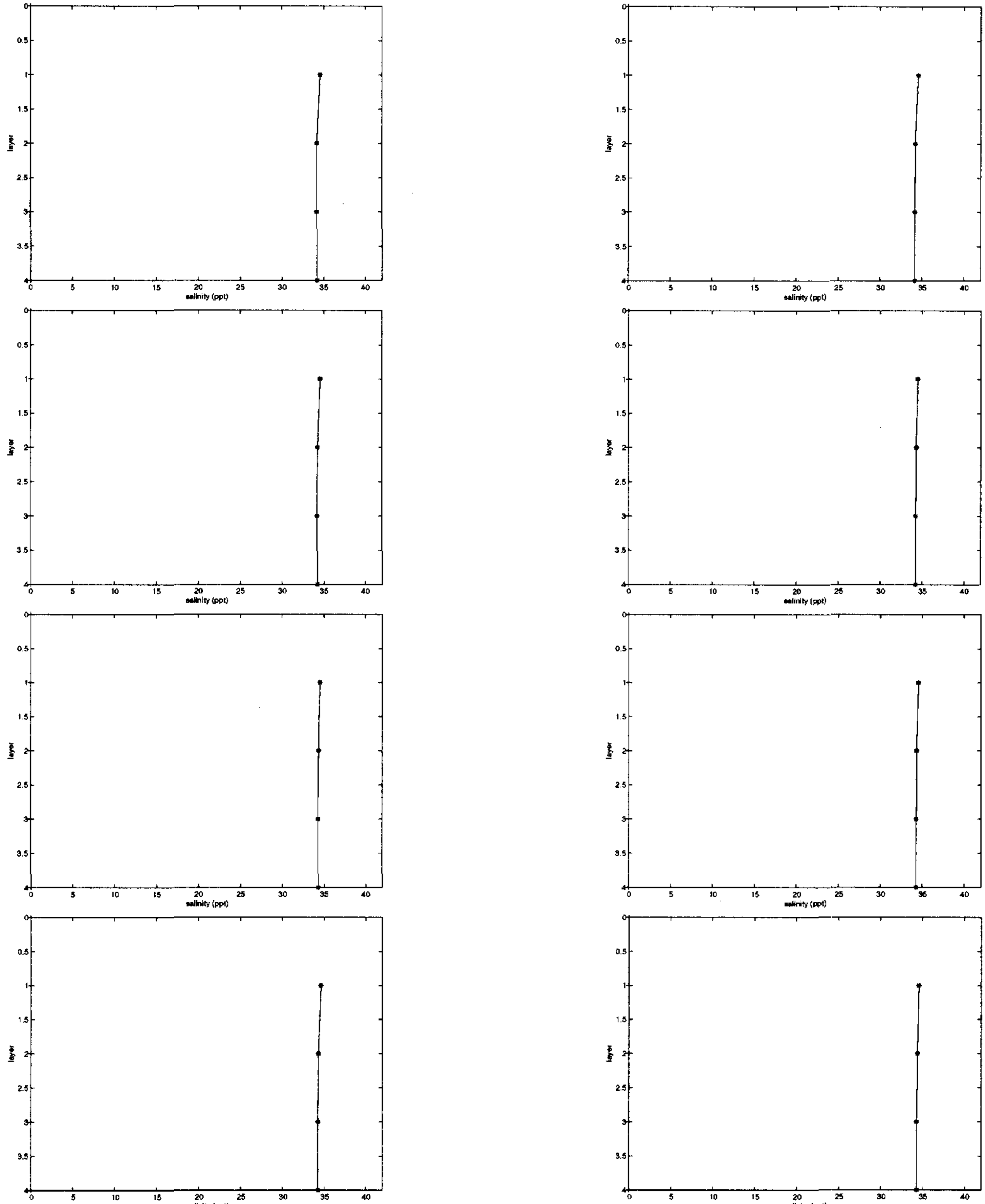

Figure 20: Vertical profiles of salinity at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at Corpus Christi Bay (location 4)

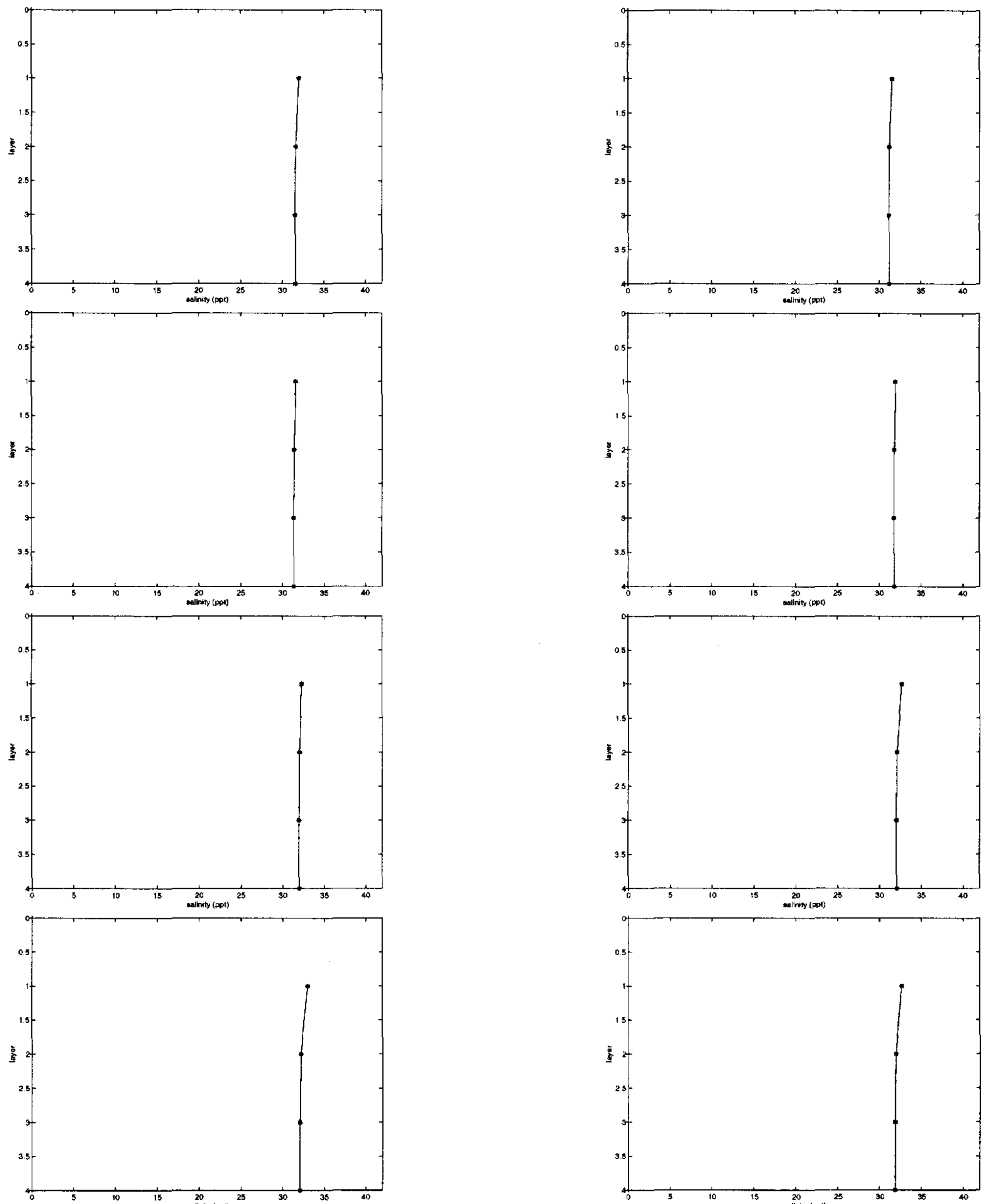

Figure 21: Vertical profiles of salinity at days 25, 25.125, 25.25, 25.375, 25.5, 25.625, 25.75 and 25.875 at Nueces Bay (location 5)

References

[1] G.L. Mellor and T. Yamada, *Development of a turbulence clsoure model for geophysical flow problems*, Reviews of Geophys. Space Phys., 20, 851-875, 1982.

[2] B. Galperin, L.H. Kantha, S. Hassid and A. Rosati, *A quasi-equilibrium turbulent energy model for geophysical flows*, J. Atmos. Sci., 45, 55-62, 1988.

[3] A.F. Blumberg, B. Galperin and D.J. O'Connor, *A quasi-equilibrium turbulent energy model for geophysical flows*, J. Phys. Oceanogr., 10, pp. 237-257, 1992.