MANUAL OF COMPUTING AND MODELING TECHNIQUES AND THEIR APPLICATION TO HYDROLOGIC STUDIES

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By

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PREFACE

This manual is written for employees of the Texas Water Commission engaged in problems of water resources, with the hope that by gaining a basic understanding of computers and models they may develop new ideas which will lead to increased efficiency and output of work. Much of the manual is devoted to explaining the basic principles of digital computers and analog computers and models mainly for the purpose of removing some of the mystery surrounding these devices as they generally appear to the layman. Other parts are concerned with hydrologic applications of advanced mathematics. The material presented is directed toward problems in both ground-water and surface-water resources; heavier emphasis is placed on ground-water applications owing to the writer's experience, however almost all material presented is from available literature. Because the aim of the manual is to present information in an understandable way, complete development is not pursued. An extensive bibliography at the end of the manual may be useful to the reader who wishes to investigate ~~his~~ area of interest more thoroughly. This presentation is not a technical paper but rather a comprehensive teaching aid, thus in order to facilitate clarity references are not documented.

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Chapter I

INTRODUCTION

Since World War II considerable progress has been made in the development of automatic computers and their applications to engineering processes, owing to advances in electronics technology. The computer field rapidly divided into two branches, digital and analog. As each computer method gained adherents considerable rivalry developed. However, this seems to be only a temporary situation, and as engineers and scientists become familiar with both methods, they generally realize that each method has its own advantages and that the choice should depend on a critical appraisal of the type of problem investigated.

Before beginning to understand the basic principles of analog and digital computers, it is advisable to point out their distinctions and differences.

First consider the <u>digital computer</u>: BASICALLY, A DIGITAL COMPUTER IS A DEVICE WHICH COUNTS. A most primitive form of digital computer is the fingers. We first learned to count by using fingers, and probably the decimal number system is an outgrowth of primitive man's use of his fingers in counting. The Chinese abacus is another primitive form of digital computer. More familiar to us are desk adding machines and calculators, complex digital computers. However, the term digital computer generally implies a more sophisticated machine called "electronic brain", the amazing device which threatens to replace so many clerical jobs and create unemployment problems. Its desirable features are tremendous speed of computation and flexibility. The important thing to remember about a digital computer (and this applies not only to the simpler ones but also to the advanced high-speed electronic digital computers) is that it deals in digits and provides rather exact numerical answers. Again, a digital computer counts. Its accuracy depends upon the number of digits provided for processing numerical data.

Next consider an analog computer or an analog model. The word analog implies similarity, or analog. Two systems are said to be analogous if there is a one-to-one correspondence in their physical properties.

Analog computers provide answers to calculations by dealing in a physical system analogous to the system which is being studied. It was pointed out in the previous paragraph that digital computers count. Correspondingly, ANALOG COMPUTERS MEASURE. An example of an analog computer is the gasoline indicator in an autombile. A float in the gasoline tank is mechanically connected to a device which controls electricity flowing into an electric meter on the dash-board. Actually electric current flow is being measured, not gasoline. But the rate of flow of electrical current is analogous to the amount of gasoline in the tank. Another example of an analog computer is the slide rule on which mathematical operations are performed by measuring lengths along the members of the rule. The term analog computer, however, generally refers to complicated devices called electronic differential analyzers; they perform mathematical operations by using laws of electricity. Their most common applications are in solutions of differential equations.

An analog model differs from an analog computer mainly in that the computer is a general-purpose device designed to solve mathematical relations, whereas a model is a special-purpose device created to simulate a physical system of interest; hence an analog model is sometimes called a simulator.

An analog computer uses physical laws for mathematical purposes, but an ANALOG MODEL HAS A DIRECT CORRESPONDENCE IN ITS PHYSICAL PROPERTIES TO PHYSICAL PROPERTIES OF ITS PROTOTYPE.

An analog computer has a broad range of utility in mathematical problems related to engineering, physics, chemistry, and so on, but an analog model is constructed only to provide answers to a specific problem.

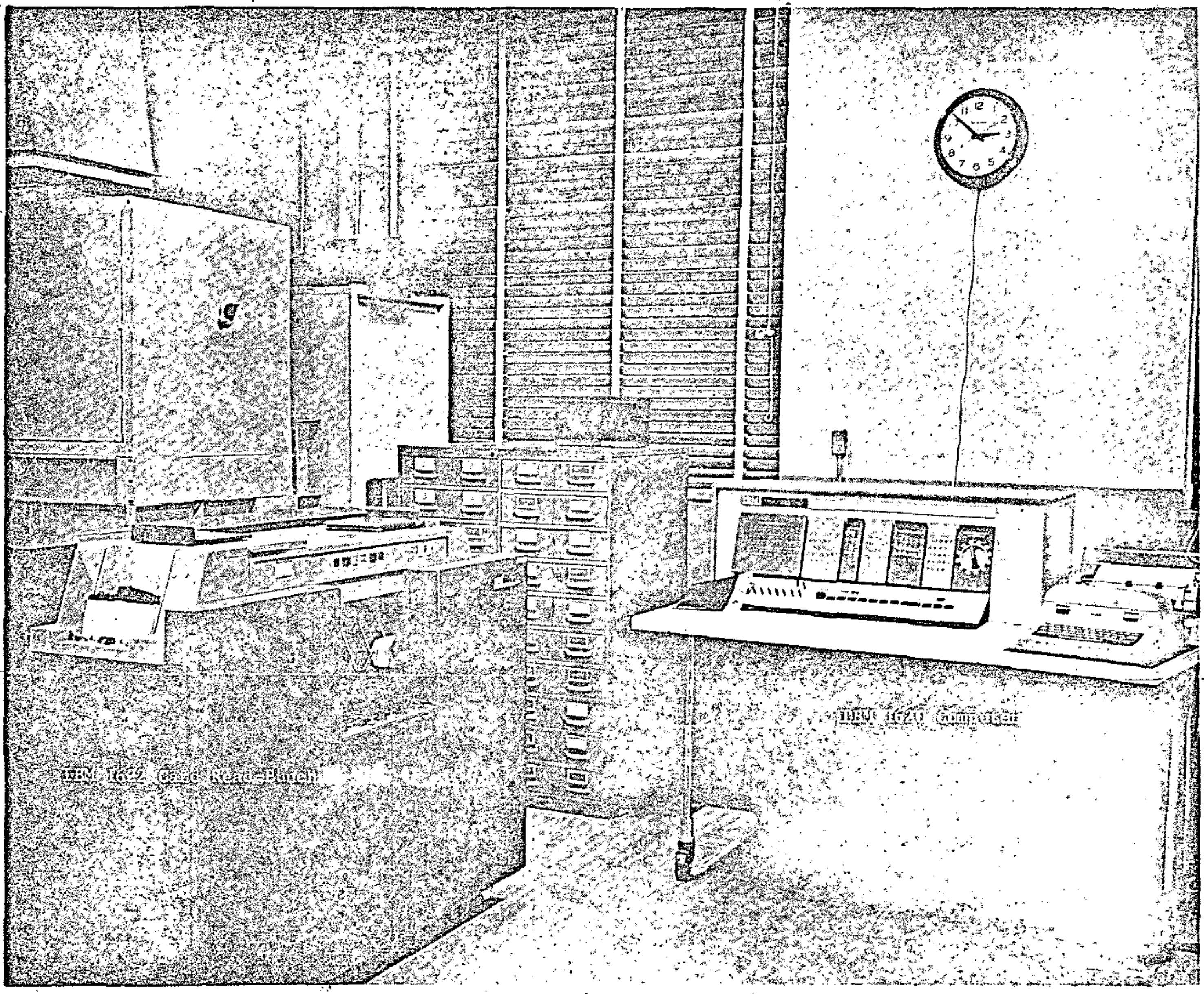
A scale model may be considered a special type of analog model. Well-publicized scale models are the river models at Vicksburg, Mississippi, where specialists of the Corps of Engineers attempt to predict, among other things, a river's hydraulic response to proposed modifications in its channel. Aircraft designers use scale models extensively in wind tunnel studies.

In scale models we observe that the physical system in prototype and model are the same. The only changes are in the scaling of dimensions. But with analog models other than scale models the physical systems are different. For example, flow of heat may be simulated or modeled by flow of electricity.

A summary of differences in analog and digital processes follow. Digital computers use counting techniques, but analog computers and models use measuring techniques. Digital computers are more accurate than analog computers and models, and where accuracy is critical, digital computers are recommended.

Computational processes in a digital computer differ from those used in manual methods and the scientist or engineer is separated from his problem during the computational phases; but with an analog computer or model the methods of computation are laws of physics, and are therefore quite direct. In this sense, an analog computer or an analog model may be distinctly advantageous in that it can provide a better insight into the behavior of a problem than a digital computer.

In the past few years notable advances have been made in digital computer technology involving greater computation speed, larger memory storage, simpler programming techniques and more efficient circuit design. Comparable advances have not been made in analog computation. Therefore many problems which previously were considered solvable only by analog methods are now being solved by digital methods. In particular there is a tendency toward preference of digital computers over analog models owing to the greater accuracy obtainable with digital equipment. A singular disadvantage in using analog models in the large investment in equipment which has application only to a specific problem.



International Business Machines Model 1620 Digital Computer
and Model 1622 Card Read-Punch

Chapter II

THE HIGH-SPEED ELECTRONIC DIGITAL COMPUTER

The high-speed electronic digital computer is often called simply a digital computer. It can perform a tremendous number of computations with incredible speed, which accounts for its acceptance in business and industry. However, because of the machine's structural complexity many are apprehensive toward developing an understanding of it. It is hoped that this chapter will overcome some of this fear by explaining generally what a digital computer is, how it works, and what must be done to use it.

First consider briefly the method of operation of a digital computer by comparing it with a desk calculator. Suppose we have numbers to be used in a computational process. Now, in using a calculator, we must decide which number to enter first, which second, and so on. We must also decide what operations (addition, subtraction, multiplication, and division) are to be used and their sequence. Computation operations are then performed in the desired sequence. The answers obtained must then be properly recorded, manually.

If we wish to use a digital computer for the same purpose, we still decide in advance which operations to use and in what sequence; furthermore we decide on the form in which the answers are to be presented. However, all these decisions are converted into instructions, called a program, which the computer reads, memorizes, and executes. The computer is also given the numbers or data to be used. The instructions and data are generally presented on punched cards or magnetic tape. The principal distinction between the calculator and the digital computer methods is in the speed of computation. With the digital computer, computation is extremely rapid, although considerable time is required to devise a program. The decision to use either a calculator or a computer should therefore be concerned with the totality of time and costs involved. A computer would be preferred for instance in problems of a recurring nature. A distinct advantage in using a computer is that by reducing the time required for tedious manual computations, morale of personnel is considerably improved, and engineers and scientists can then devote more time to technical duties.

A digital computer can add algebraically, make decisions, and follow a logical set of instructions. The method in which a computer solves a problem is to read and store instructions, read and process data according to the instructions, and present the results in the desired form.

Having considered the computer's method of operation we now investigate its mechanism. The mechanism of a desk calculator will be compared with the mechanism of a digital computer.

Anyone familiar with the desk calculator realizes that numbers appearing in the register windows of the calculator are actually digits printed on individual cylinders. Each cylinder contains 10 digits, from 0 to 9 (this is basically the decimal number system). These cylinders are turned by mechanical pulses generated in the machine's computing mechanism. A high-speed digital computer uses electronic pulses rather than mechanical pulses, and in the more advanced computers millions of pulses are generated each second. These pulses magnetize tiny iron cores and open and close tiny electronic switches connected in intricate combinations. Some of these tiny components are used to store information (the memory), others are used in computing processes.

A digital computer does not use the decimal number system; it uses the binary number system, which contains only 2 digits. The reason for this is that in electricity and magnetism there are only 2 stable states. Electricity or magnetism is either positive or negative. An electric switch is either on or off.

A comparison of some decimal numbers and corresponding binary numbers are shown below:

Decimal System	Binary System
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011

In manual computations the binary system is much more cumbersome than the decimal system, and it would seem impractical to use such a system. It presents no problem, however, to an electronic digital computer, because of the speed in which the binary system can be handled. Digital computers are designed with principles of <u>boolean algebra</u>, an abstract algebra, which deals with two sets.

Although hundreds of thousands of electrical parts are used in an advanced digital computer, they can be packed into a relatively small space, owing to recent advances in miniaturization techniques in electronics.

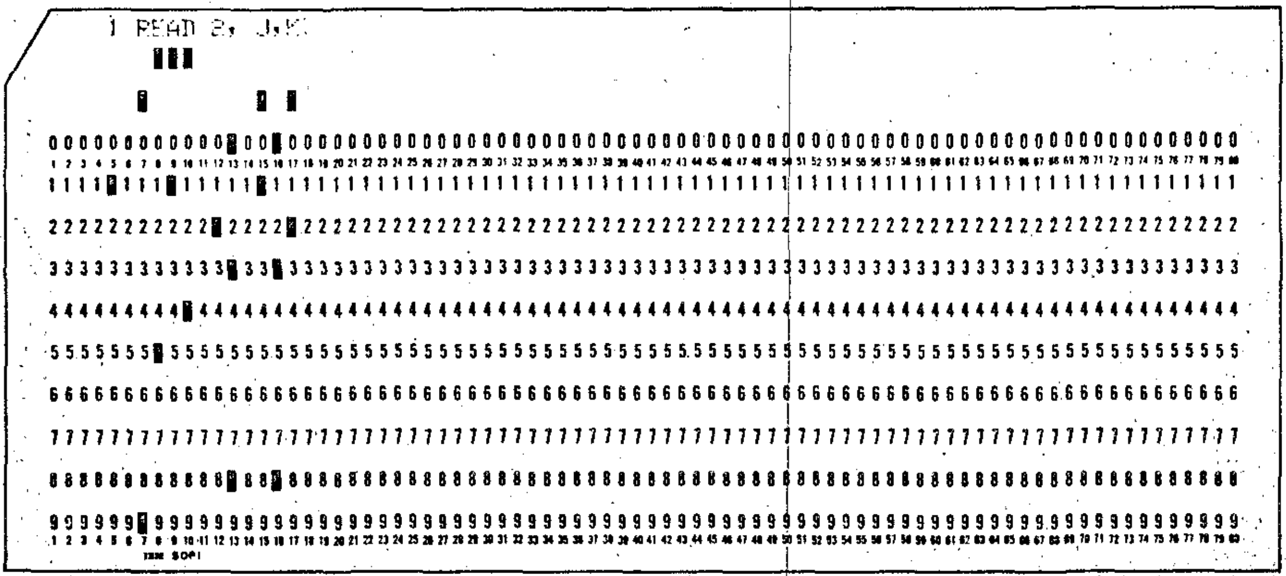
One would suppose that in order to be able to use a digital computer he would have to be acquainted with binary arithmetic and boolean algebra. However, the computer design engineers have assisted us by placing between the operator and the computer a "black box", called a compiler, which translates data from decimal to binary system and codes our "human" instructions into machine language. Because information must be presented to the compiler in a standard form, there have been evolved many compiler languages. Some compiler languages require a great deal of knowledge about the computer's system, but there has been recently developed for scientific and engineering work a rather straightforward language called FORTRAN (FORmula TRANslation). One of the principal advantages of FORTRAN is that besides being easily learned it has become so standard that now almost all computers for scientific application have FORTRAN compilers.

One disadvantage of FORTRAN is that it cannot utilize all of a computer's capability. By being removed from the absolute computer language, and being dependent on a compiler language, some of the computer's resources are sacrificed. A FORTRAN compiler is to a digital computer what an automatic transmission is to an automobile. The automatic transmission is easily learned and is entirely satisfactory for most driving purposes, yet no thinking driver would enter the Pike's Peak Race with a car so equipped. There are some tasks for which an automatic transmission is not as adequate as a standard transmission. Similarly, although many problems can be solved on a digital computer by using FORTRAN there are also some problems which can not be solved unless the absolute machine language or a compiler language dealing more intimately with the machine than FORTRAN is used.

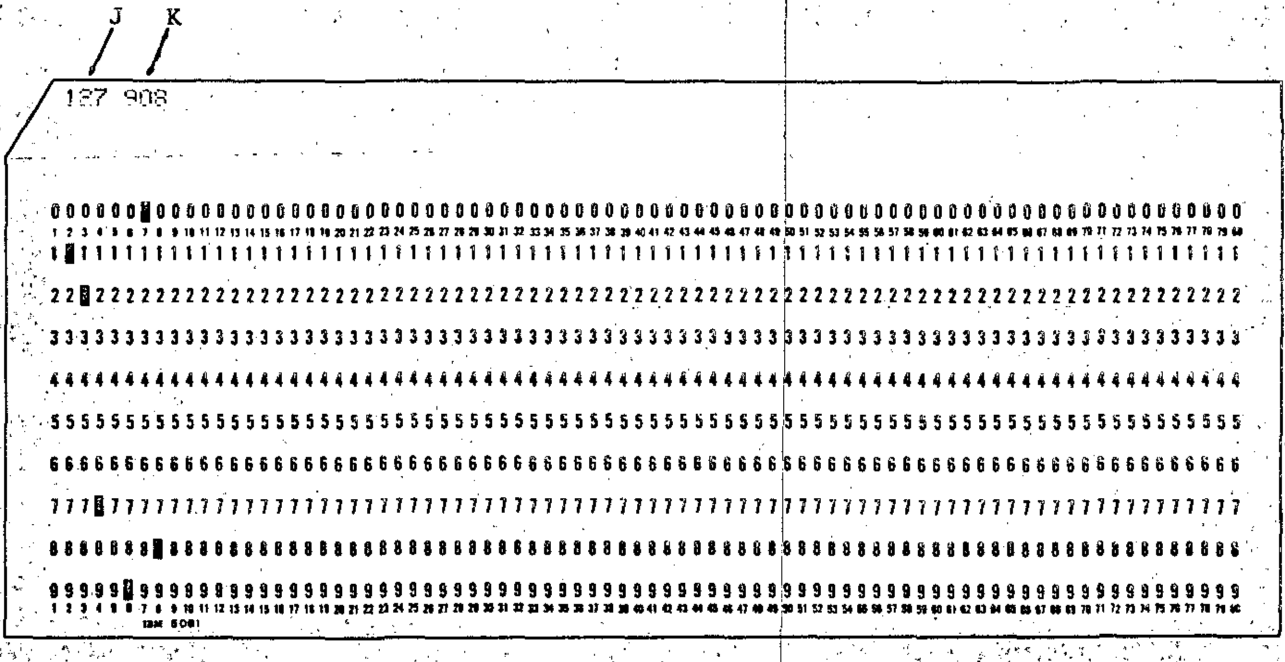
In general, the difficulty of creating a program depends on the complexity of the problem. Many problems can become so enormous and involved that the services of a professional programmer are warranted. However if the results are important enough, the cost of the services are justified.

Most computers have built into them special <u>subroutines</u>, which are "prefabricated" instructions for commonly used mathematical operations such as square roots, trigonometric functions, and logarithms. A simple code in a program calls the desired subroutine into action.

An elementary example of solving a problem by the use of a digital computer using FORTRAN language is shown below. The program can be used as often as desired - it is not specific. If the same operation with different data is to be performed later, only the data needs to be changed. The program instructs the computer to calculate the sum of two whole numbers which have no more than



Punched Card for FORTRAN Statement No. 1 in Example Program



Data Card for Numbers J and K in Example Program three digits each. Each statement is to be punched on a separate card. (The punched card for statement 1 is illustrated on the facing page.)

```
1  READ 2, J, K
2  FORMAT (2I4)
3  L = J + K
4  PRINT 5, L
5  FORMAT (4X, I5)
6  GO TO 1
7  END
```

Statement 1 instructs the computer to read from a data card two numbers, which it will call J and K, according to specifications of statement 2.

Statement 2, a format statement, specifies that J is found punched within the first four columns of a data card (the first column is for a plus or minus sign), and that K will be found punched within the second four columns. The letter I is a code which signifies that the numbers are integers - decimals are not used. (The punched card for data in this format is shown on the facing page.)

Statement 3 instructs the computer to calculate the sum of J and K and to call the result L.

Statement 4 instructs the computer to print the number called L, in accordance with the format specifications of statement 5.

Statement 5 specifies that the sum (L) will be printed in a block 5 spaces wide, located 4 spaces to the right of the left hand margin.

Statement 6 directs the computer to return to statement 1 and read another data card, and hence repeat the addition and print-out process.

Statement 7 is necessary for the proper reading and compiling of instructions. It is a signal to the computer that there are no more instructions and to proceed with the execution of the problem.

Digital computers are quite expensive and are generally purchased only by large corporations. Smaller corporations either use computer consulting servic or obtain computers on a rental basis. Computer-time rental rates range from about $35/hr. for the smaller computers such as Control Data Corporation Model 160A to about $550/hr. for the largest computers such as the International Business Machines Model 7090.

Chapter III

ANALOG MODELS

Before proceeding with the discussion on analog models it will be helpful to review differences in the digital and analog systems. It was pointed out that the principal difference between digital and analog processes was that digital types use counting techniques, whereas analog types use measurements of physical quantities. Furthermore the accuracy obtainable with analog equipment is limited by the quality of its components.

There are, in general, two kinds of analog systems: analog models (simulators) and analog computers (differential analyzers). In an analog model, a prototype physical system, such as a ground-water aquifer, is represented or modelled by another physical system with similar properties, such as an electric circuit. The model is subjected to conditions analogous to those encountered in the prototype, and the behavior or reaction of the model is studied and/or measured.

An analog computer has features similar to an analog model. However an analog model is built only for a specific problem, but an analog computer is a general purpose tool designed for a wide variety of applications. Analog computers are used extensively in engineering problems to solve ordinary differential equations and are discussed more specifically in Chapter VI.

Analog models are used in many branches of engineering research. The choice and design of the model depends on the type of problem, the results sought, and economic and other factors. A familiar example of a special type of analog model is the scale model. Both prototype and model have the same physical properties; only the dimensions differ. Where the physical systems differ, electric- or mechanical-analog models are generally used.

An example of a scale model is a sand tank, in which a glass box is filled with sand, and provision made for water to flow continuously through the sand. Various conditions may be imposed, for instance water may be pumped from a simulated well, and changes in configuration of the water levels in the sand observed. Often this type of model is used only to illustrate the behavior of the prototype system; measurements can be misleading.

An example of a mechanical-analog model is a membrane. To illustrate an application, consider a well pumping from a water-table aquifer. Water moves through sand toward the well causing the water table to form an inverted cone with its apex at the well. The same shape occurs in a membrane when one presses a sharp instrument against it. Elastic properties of the membrane are analogous to water-transmitting properties of the aquifer, and the magnitude of the point force applied to the membrane is analogous to the rate of pumping from the aquifer.

Electric-analog models, however, appear to have greater utility than mechanical-analog models because electric properties have analogs in so many other physical systems. Furthermore, electric components are relatively inexpensive, their properties are not affected by gravity, substitutions or modifications are easily accomplished, and an electric system can be constructed for convenient access in measuring. Specific discussion of analog models in this manual is confined to electric types.

Construction of an electric-analog model requires that one be familiar not only with the physical behavior of the prototype system but also with electronic theory and technique in order that parts for the model can be properly selected and assembled. The general procedure is to construct the model (the system), apply an external excitation, and observe or measure the response.

Electric-analog models are used in two types of engineering problems, synthesis problems and analysis problems. A synthesis problem is one in which the excitation and response are specified and it is desired to determine the system conforming to these specifications. An analysis problem, on the other hand, is one in which the system and excitation are specified and it is desired to determine the response.

In a synthesis problem the usual procedure is first to construct a model by estimation of properties and to test the model's response. Then the model is repeatedly modified and tested until its response conforms with sufficient accuracy to the desired specifications.

Electric-analog models provide solutions to partial differential equations, a subject which will be developed in the next chapter.

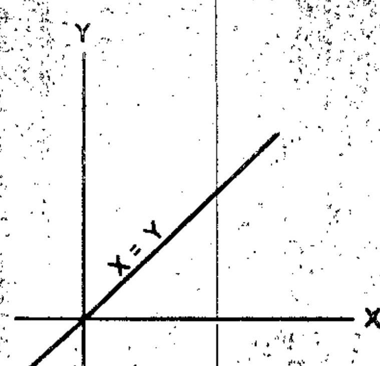
Chapter IV

PARTIAL DIFFERENTIAL EQUATIONS

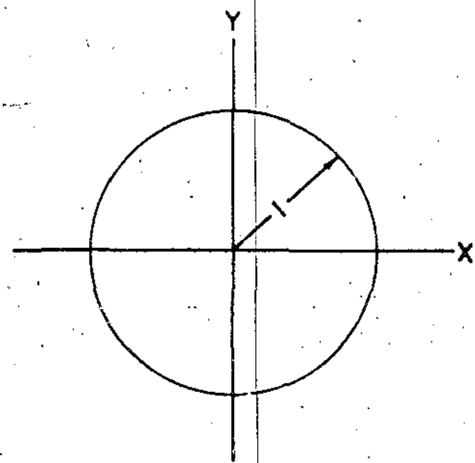
Partial differential equations are commonly used in physical sciences and engineering; mathematical methods of handling them have principally been an outgrowth of scientific research. It is the purpose of this chapter to attempt only a physical explanation of partial differential equations; mathematical precision is relaxed. Moreover, in many cases partial differential equations cannot be solved by classic mathematical techniques, but only by approximation methods. Nevertheless, the mathematical concepts are useful for gaining a thorough understanding of the subject. Simplified explanations of two commonly-used types of partial differential equations are given in this section.

Before introducing partial differential equations it may be helpful to point out some elements of their broader mathematical field, calculus.

First observe that an algebraic equation in two variables (for example, x and y) can be displayed in graphic form. Some equations graph as lines, other as curves. For instance the equation x = y graphs as a line,



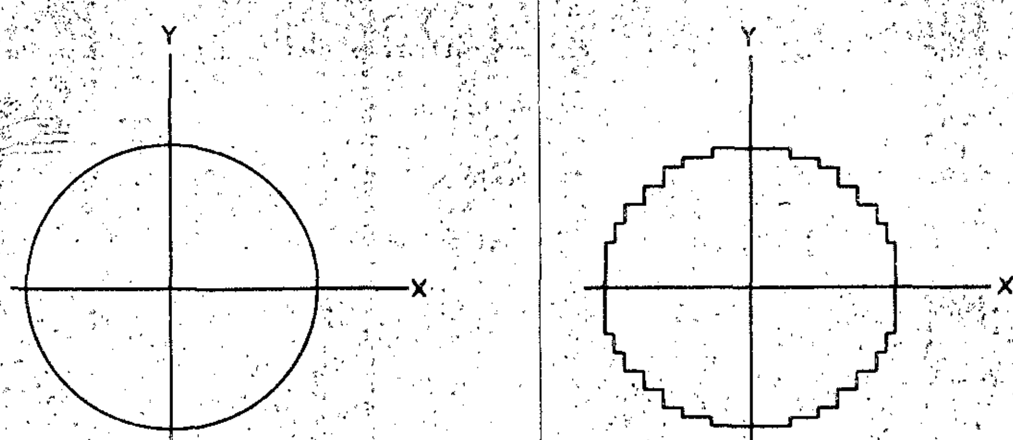
and the equation $x^2 + y^2 = 1$ graphs as a circle.



In calculus, where equations graph as curves, the curves are considered to be composed of tiny line segments joined end to end. Formally, calculus problems investigate equations by analyzing their graphs as the lengths of tiny line segments tend to zero.

However, in many engineering problems only a graphical curve form is known; the associated equation is not available. In such problems, satisfactory solutions to problems may be obtained by approximating the curve by line segments of finite length. (The smaller the line segments, the more accurate the approximation.) Such methods of solution are called finite difference methods.

An elementary example of a finite difference method is to calculate the area of a circle by approximating its boundaries as straight line segments.



The circle on the left is approximated by the figure on the right whose area can be calculated by "counting squares".

Finite difference methods can also be used to approximate algebraic equations in three variables, which graph as surfaces in space.

Differential equations is a discipline which deals with differential calculus. Differential calculus deals with expressions called derivatives. A derivative of an algebraic equation describes the inclination or slope of any tiny line segment in the graph of the curve. Solving a differential equation consists of finding an algebraic equation whose derivatives satisfy the differential equation.

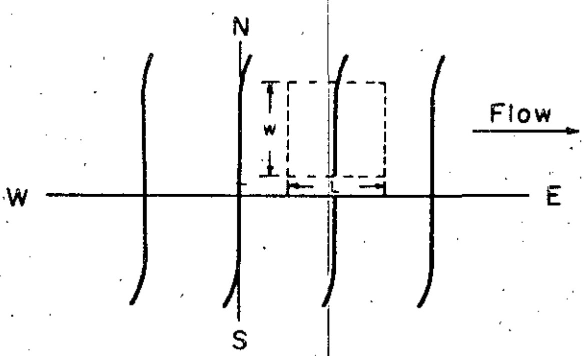
Partial differential equations involve algebraic equations in more than two variables. They are composed of partial derivatives, which are derivatives involving only two of the variables. Partial differential equations are encountered frequently in problems involving space and/or time.

We begin with Laplace's partial differential equation,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0.$$

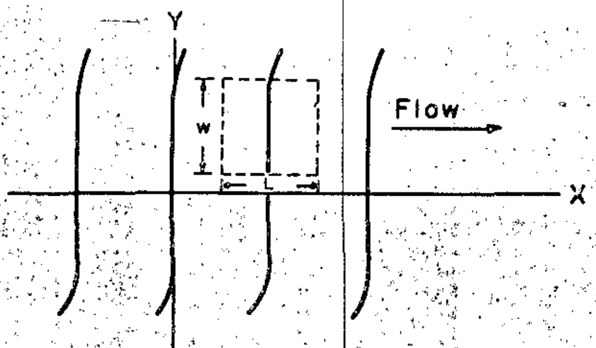
Expressed simply, Laplace's equation means that the rate of inflow equals the rate of outflow. If we deal with a hydraulic system in which the rate of movement of water flowing into the system is the same as the rate of movement of water flowing out of the system, then we say Laplace's equation is satisfied. Furthermore, there is no change in the amount of water stored in the system. To cite a common example, if you spend all your monthly income and your savings remain intact, then your financial system more or less obeys Laplace's equation.

In order to see how Laplace's equation applies, consider a water-level contour map:

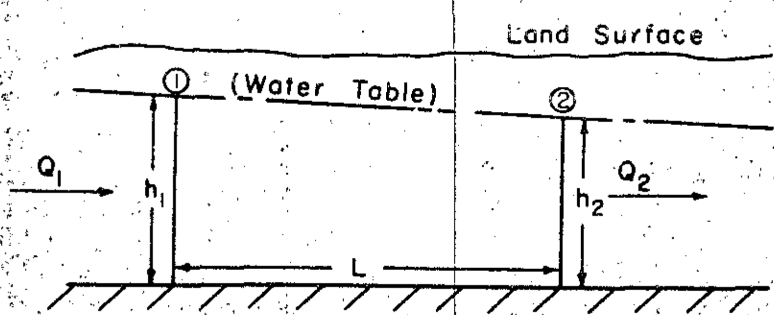


The flow is clearly in one principal direction, east. Water enters the west side of the small outlined box; and exits from the east side. The box has width, W, and length, L, as indicated.

Next, change the map directions E and N to graph directions, x and y.



Now, consider a cross section from left to right across the small box, along the path of flow.



At point 1:

the rate of inflow is Q_1,

the piezometric head is h_1,

the hydraulic gradient is I_1.

At point 2:

the rate of outflow is Q_2,

the piezometric head is h_2,

the hydraulic gradient is I_2.

If the coefficient of transmissibility of the sand is T,

the INFLOW at point 1 is $Q_1 = TI_1W$,

the OUTFLOW at point 2 is $Q_2 = TI_2W$.

Since INFLOW - OUTFLOW = 0,

$$TW(I_1 - I_2) = 0,$$

and

$$I_1 - I_2 = 0.$$

Divide both sides of the last expression by L, the length of the flow-path in the small box,

(1) $$\frac{I_1 - I_2}{L} = 0.$$

Hydraulic gradients, I_1 and I_2, are expressed in mathematical symbols as:

$$I_1 = \frac{-\partial h_1}{\partial x},$$

$$I_2 = \frac{-\partial h_2}{\partial x}.$$

The terms on the right are partial derivatives (slopes). Now substitute these terms in equation (1) to obtain

$$\frac{\frac{\partial h_2}{\partial x} - \frac{\partial h_1}{\partial x}}{L} = 0.$$

When L becomes very tiny, we have the second partial derivative which is, in symbol form,

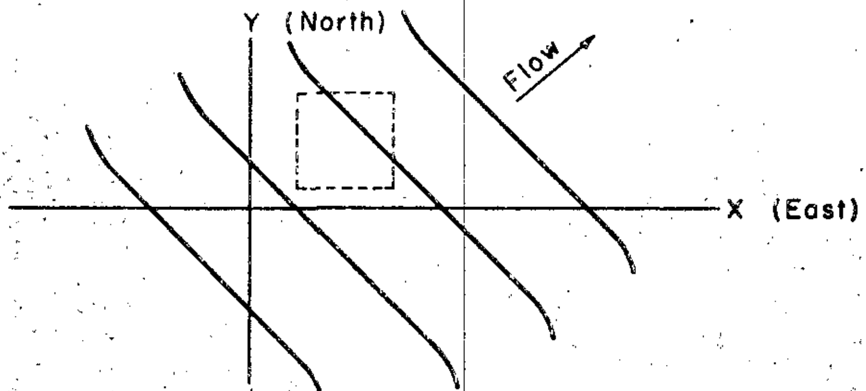
$$\lim_{L \to 0} \frac{\frac{\partial h_2}{\partial x} - \frac{\partial h_1}{\partial x}}{L} = \frac{\partial^2 h}{\partial x^2}.$$

The expression

$$\frac{\partial^2 h}{\partial x^2} = 0$$

is Laplace's equation for flow in one dimension.

Now consider another water-level contour map.



The flow is now in two principal directions, north and east. Water enters the small box on the south and west sides and exits on the north and east sides. By using a process similar to obtaining Laplace's equation for flow in one dimension we can obtain Laplace's equation for flow in 2 dimensions:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0.$$

Suppose now that water also flows downward (in the z-direction) in which case there are three directional components of flow. Laplace's equation for flow in three dimensions is

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0.$$

Sometimes Laplace's equation appears in terser symbolic form:

$$\nabla^2 h(x) = \frac{\partial^2 h}{\partial x^2} = 0$$

$$\nabla^2 h(x,y) = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

$$\nabla^2 h(x,y,z) = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0.$$

The symbol ∇^2 is the Laplacian operator.

The important thing to remember is that whenever a fluid system satisfies Laplace's partial differential equation (steady state equation), then at any place in the system the rate of inflow equals the rate of outflow, and there is no change at any time in the amount of fluid stored in the system. Furthermore, the hydraulic head at any point does not change with the passage of time, and there are no fluid losses or gains within the system.

Returning to the water level contour maps recall that we considered only the internal part of a flow system. Because there was a flow condition fluid obviously had to enter and leave the system at places external to it. These places are called boundaries, and the description of the fluid system becomes more meaningful if the location of boundaries and the piezometric head or other conditions there are designated. Hence, a flow system which is described by a partial differential equation and boundary conditions is sometimes called a boundary value problem.

Laplace's equation is generally of little benefit in our quantitative ground-water studies. Its greatest utility is in problems of drainage and seepage - where steady-state flow conditions occur. In quantitative ground-water studies, we are usually interested in non-steady flow, wherein piezometric head (or water level) at any point in a system changes with passage of time, and consequently storage of water in the system continuously changes. A simple general equation describing non-steady flow is:

INFLOW - OUTFLOW = CHANGE OF STORAGE.

Think about this for a few minutes. It is indeed logical and is applicable not only to ground-water systems but also to surface-water reservoirs.

We alter this equation slightly to consider passage of time:

RATE OF INFLOW - RATE OF OUTFLOW = RATE OF CHANGE OF STORAGE.

Now let's return to Laplace's equation in two dimensions,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0.$$

Recall that we obtained this after dividing an expression by the coefficient of transmissibility, T. Then it is permissible to multiply both sides of this equation by T:

$$T\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) = 0.$$

This is a more meaningful statement that inflow and outflow rates are identical, and the left side of the equation is in fact a statement of the difference in rates of inflow and outflow.

But in the non-steady state the difference between the rate of inflow and outflow will not be 0 but will be

(2) $$T\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) = \text{RATE OF CHANGE OF STORAGE.}$$

To express rate of change of storage mathematically, first consider the aquifer property of storage. Coefficient of storage, S, is defined as the volume change of water in storage over a unit area as a result of a unit change in hydraulic head, or

CHANGE OF STORAGE = (CHANGE OF HEAD) x S,

and

(3) RATE OF CHANGE OF STORAGE = (RATE OF CHANGE OF HEAD) x S.

Now

(4) $$\text{RATE OF CHANGE OF HEAD} = \frac{\partial h}{\partial t},$$

the first partial derivative of head with respect to time.

Combining equations (2), (3) and (4),

$$T\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) = S\left(\frac{\partial h}{\partial t}\right).$$

Divide both sides by T,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\left(\frac{\partial h}{\partial t}\right).$$

This is the non-steady state flow equation in 2 dimensions. Using Laplacian operator notation,

$$\nabla^2 h = \frac{S}{T}\left(\frac{\partial h}{\partial t}\right).$$

The non-steady state equation is commonly called the continuity equation, or the diffusion equation, so named because of its application to heat flow in solids.

A system which satisfies the non-steady state, or continuity equation is called a transient system because changes in the system with time are considered.

It may be of interest to note that mathematicians classify the Laplace and the diffusion equations respectively as elliptic and parabolic types of partial differential equations. The classification is based on similarity of appearance of differential equations and algebraic equations of analytic geometry. The classication does not reflect shapes in flow patterns or boundaries.

In summary the two important differential equations we have considered for water-resources studies are:

1. Laplace's equation (steady state equation),

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0,$$

which specifies that anywhere in the flow system

a. RATE OF INFLOW = RATE OF OUTFLOW

b. no change in storage occurs.

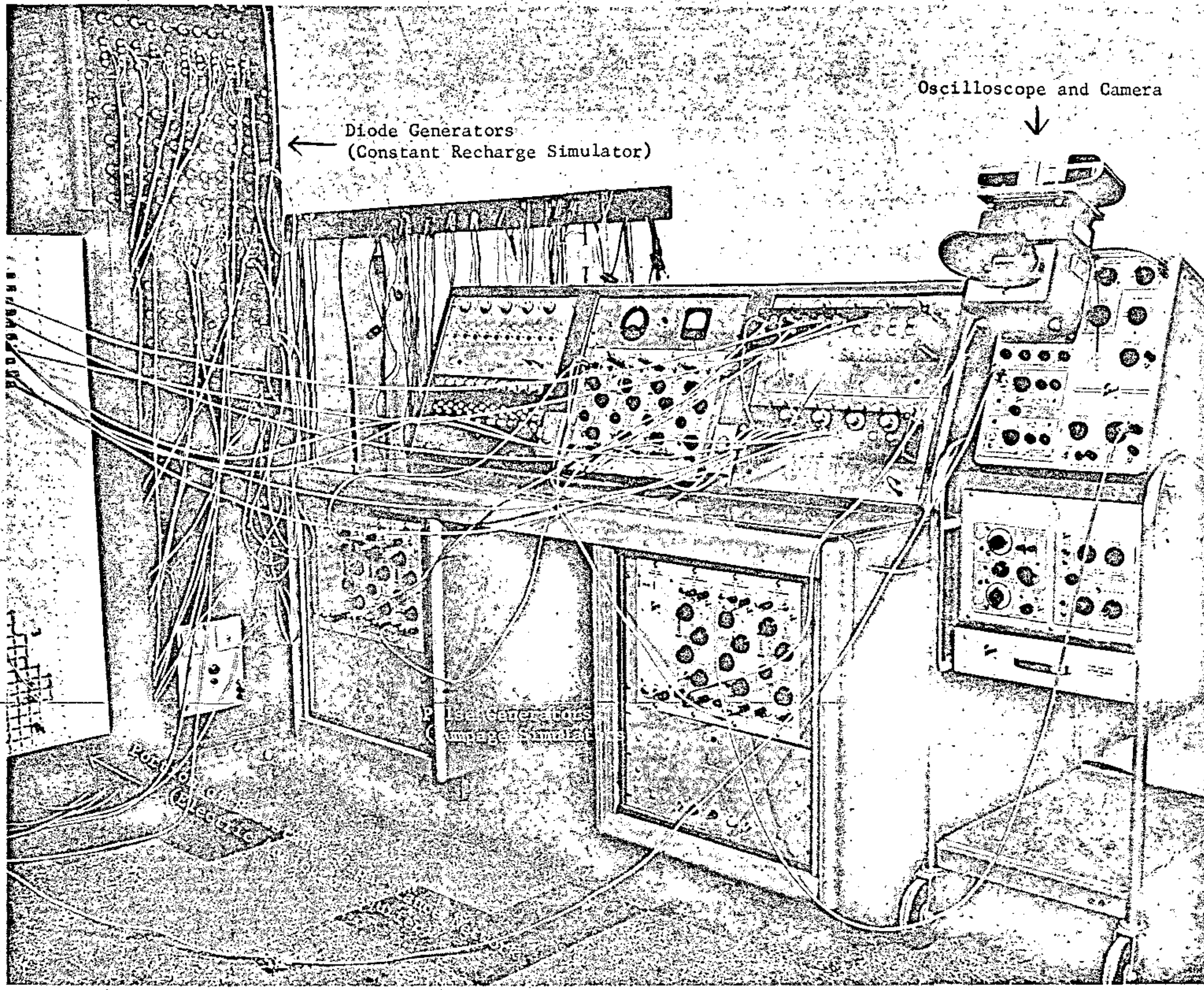
2. The continuity equation (non-steady state equation),

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\left(\frac{\partial h}{\partial t}\right),$$

which specifies that

RATE OF INFLOW - RATE OF OUTFLOW = RATE OF CHANGE OF STORAGE.

It is strongly emphasized that the term fluid movement used herein refers to bulk rate of flow, such as gallons per minute or cubic feet per second; it does not mean the discreet velocity of individual fluid particles such as feet per day. The reason for this distinction will be shown in the following chapter which describes the expression of fluid-flow systems with electric-analog models.



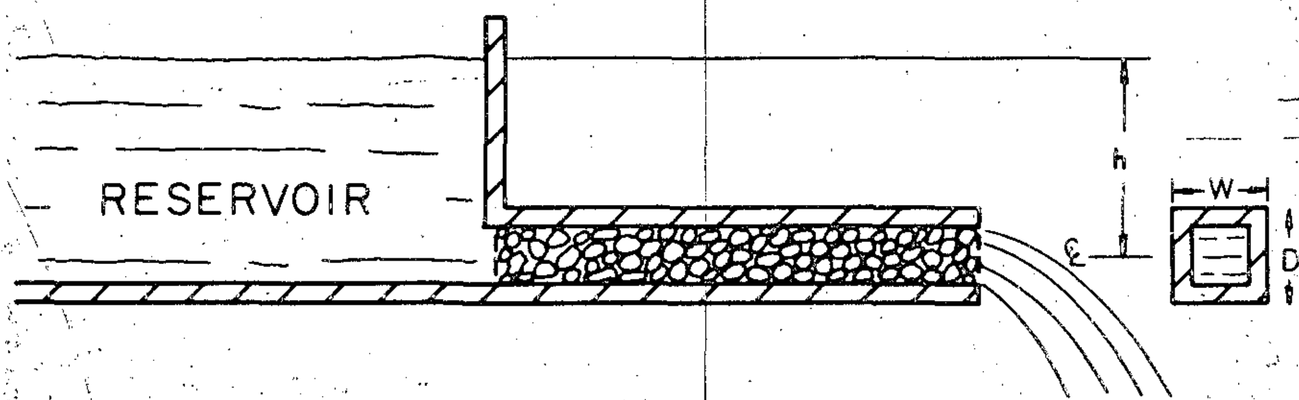
Equipment Used in Electric-Analog Model
Study of Aquifers in Houston District

Chapter V

FORMULATION OF HYDROLOGIC PROBLEMS INTO ELECTRIC-ANALOG MODELS

This chapter presents an explanation of some of the analogous properties of flow and storage in fluid and electric systems.

First, considering flow properties, we begin with a special case of fluid flow through a pipe.



Fluid flows from left to right through the square gravel-filled pipe shown because there is a difference in pressure from left to right, along the flow path; water flows in the direction of least pressure. The rate of flow, Q, depends in part on the difference in pressure or piezometric head, h. The gravel-filled pipe offers resistance to flow; the amount of resistance depends on cross sectional area, length, and permeability of the gravel. The more resistance there is to flow the less the rate of flow will be. The rate of flow through the pipe can be expressed by the following equation:

$$Q = P\left(\frac{h}{L}\right)WD = T\,\frac{W}{L}h = Yh,$$

where Q is the rate of flow through the pipe,

P is aquifer coefficient of permeability (applied to the gravel in the pipe),

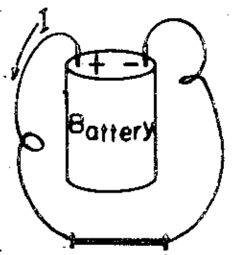
T is aquifer coefficient of transmissibility (applied to the gravel in the pipe), and

Y is hydraulic conductance (Y = TW/L).

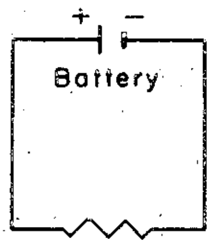
In words,

RATE OF FLOW = HYDRAULIC CONDUCTANCE x HEAD DIFFERENCE.

Next let us compare the corresponding properties of flow of electricity through a rod, or resistor, as shown by the following illustrations and schematic diagram.



ROD (resistor)



Resistor

Similar to the flow of fluid in a pipe, electric charges (coulombs) flow from left to right through the rod because there is a difference in electric pressure, or potential, between the ends of the rod. The direction of flow is from higher to lower potential, and the more resistance offered by the rod to the flow of charges the less will be the rate of flow, I. The rate of flow of electric charges is current, measured in amperes, or coloumbs per second. The appropriate equation of flow of electricity through the rod is Ohm's Law:

$$I = \frac{1}{R} V = KV,$$

where

I = current, in amperes,

V = difference in electric potential, in volts (between the ends of the rod)

R = electric resistance (of the rod), in ohms,

$K = 1/R$ = electric conductance.

Corresponding analogies between the fluid and electric systems discussed are presented below.

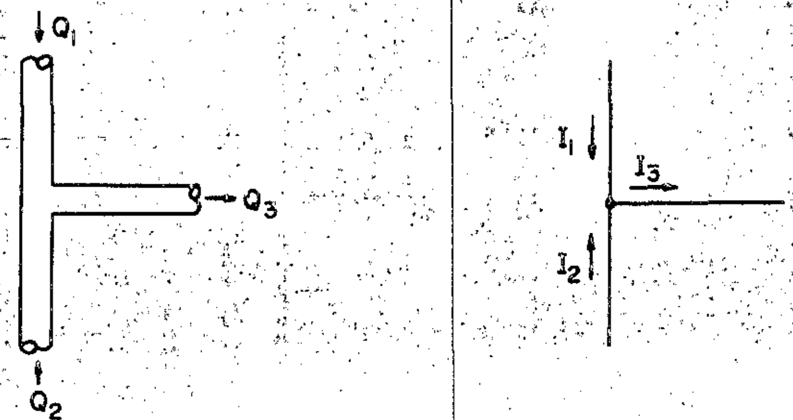
Fluid ($Q = Yh$)	Electric ($I = KV$)
h, hydraulic potential or piezometric head, in feet	V, electric potential, in volts
Y, hydraulic conductance, in gpd/ft	K, electric conductance, in mhos
Q, volume rate of flow, in gpd	I, electric current (rate of flow of electric charges), in amperes

With these corresponding properties one can use an electric analog in order to obtain fluid-flow-rate.

$$Q = c_q KV$$

where c_q is a scaling factor or constant of proportionality.

Another flow property common to both fluid and electric systems is illustrated below.

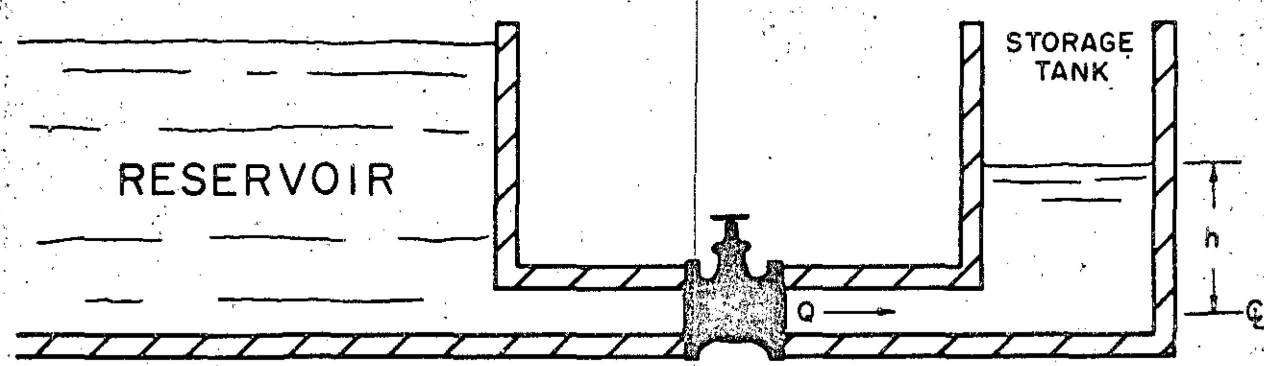


The electric system shown on the right illustrates Kirchoff's Current Law, which states that the sum of currents flowing into a junction equals the sum of currents flowing out of the junction.

An important property of fluid flow which does not have a corresponding analog in electric flow is that of particle velocity. A fluid particle can have practically any velocity, but the absolute velocity of an electric charge is the speed of light. Therefore only bulk rate of flow, not particle velocity, in fluid systems can be simulated directly by electric systems.

We now direct our attention to properties of storage in fluid and electric systems, and begin with fluid systems.

Consider the diagram below, in which fluid flows out of a large reservoir into a small storage tank when a valve between the reservoir and the tank is opened.



When the valve is opened fluid flows into the storage tank and the fluid level in it rises. The amount of fluid in the storage tank at any time after the valve is opened depends upon the size of the tank and the height, h, of the fluid level in it. It is intuitive that the rate of filling is greater at first when the difference in head between the reservoir and the tank is greater; or rate of flow, Q, through the connecting pipe decreases as the fluid level in the tank moves closer to the fluid level in the reservoir. Furthermore, as h increases, the amount of fluid stored in the tank increases. An equation which expresses the rate of change of storage in the tank is:

RATE OF CHANGE OF STORAGE = A x (RATE OF CHANGE OF HEAD) = Q.

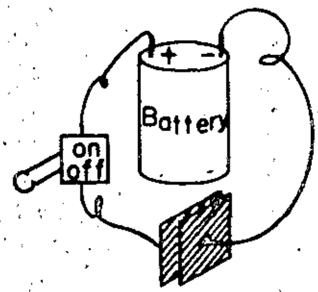
where A is the cross sectional area of the tank. In symbol form,

$$Q = A\frac{\partial h}{\partial t}.$$

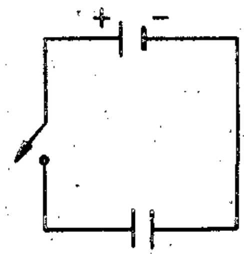
If the tank were filled with sand or gravel having coefficient of storage, S, then

$$Q = AS\frac{\partial h}{\partial t}.$$

Similar properties occur in electric storage. Consider the sketch below and the accompanying schematic diagram.



Capacitor



Capacitor

When the switch is turned on, positive electric charges flow onto the capacitor plate on the left, because of the attraction by the negative charges on the other plate. As the positive charges accumulate on the left plate there is a positive potential buildup; that is the positive voltage voltage, V, on the left plate becomes increasingly greater in time, approaching the potential of the positive battery terminal. At any instant, the amount of charge stored depends on the area of the plate (capacitance) and its electric potential, V. However as the potential increases, the charges accumulate at progressively slower rates since the attraction owing to negative charges on the right plate is progressively neutralized.

The storage expression for electricity can be described as follows:

RATE OF CHANGE OF STORAGE OF CHARGE = (CAPACITANCE) x (RATE OF CHANGE OF POTENTIAL), or

$$I = C \frac{\partial V}{\partial t},$$

where C is capacitance, in farads.

We now compare analogous storage properties in fluid and electric systems. In the fluid flow system described, the rate of change of storage was Q, in gallons per day. In the electric system described, the rate of change of storage was I, in amperes, which is also the rate of change of storage of charge. A capacitor stores electrical charges in a manner similar to that in which a container stores fluid. Hence, Q is analogous to I, and AS is analogous to C. A capacitor, then can be used to simulate aquifer storage properties. Using the above relationships, an equation for an electric simulator of fluid storage would be

$$\text{RATE OF CHANGE OF FLUID IN STORAGE} = c_s C \frac{\partial V}{\partial t},$$

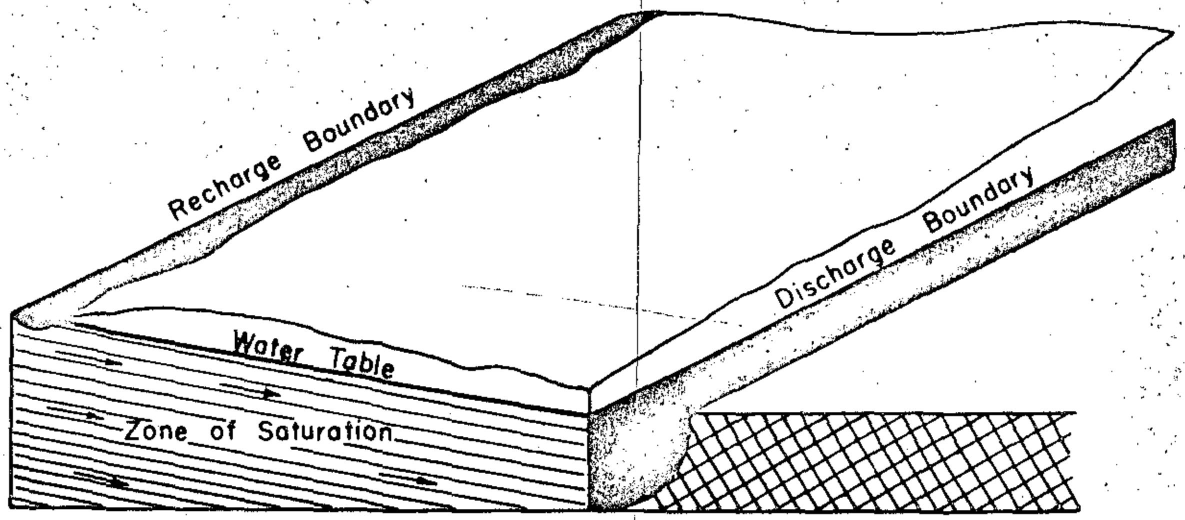
or

$$AS \frac{\partial h}{\partial t} = c_s C \frac{\partial V}{\partial t},$$

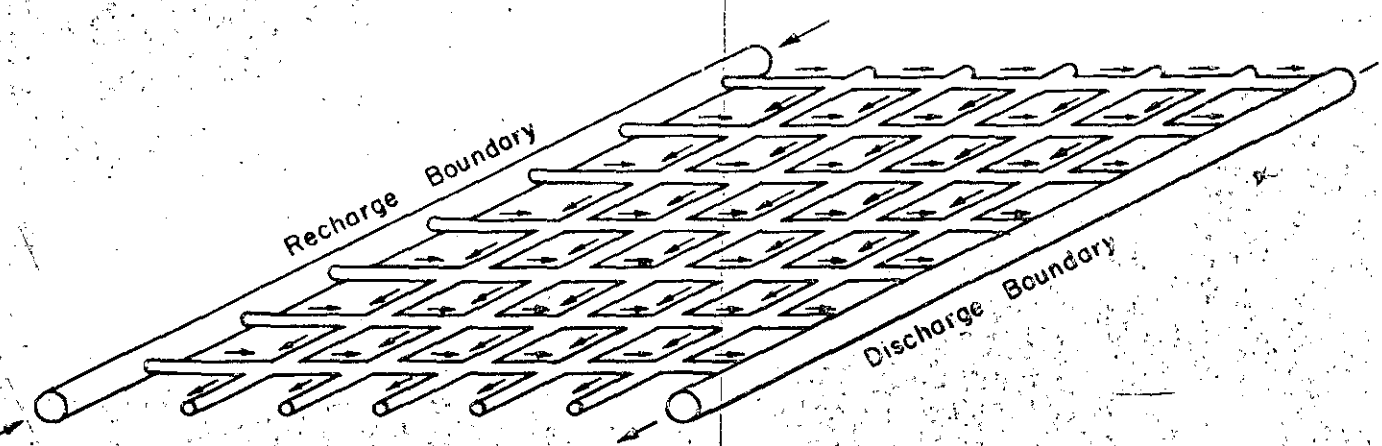
where c_s is a scaling factor.

It may be helpful at this point to review previous discussions. First, fluid-flow fields and the corresponding partial differential equations which describe these field were explained. Then fluid flow in pipes was compared to electric flow in conductors (or resistors), and fluid storage was compared to electric storage. The problem of how to present a fluid-flow system by means of an electric-analog model is still unexplained. The procedure is first to express the flow field as a pipe network, and then to represent the flow of water through the pipe network by the flow of electricity through a network of electric components.

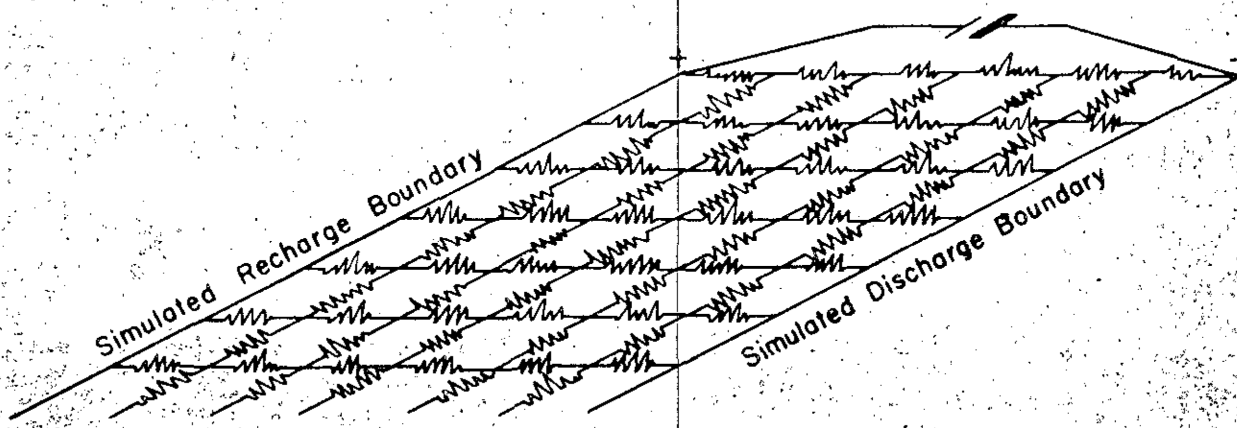
To demonstrate this, consider a steady-state flow system, or aquifer, which is describable by Laplace's equation and boundary conditions. If the flow field is an aquifer it might be like this.



Now suppose a pipe network is constructed such that boundary conditions, head, and general direction of movement in the pipe network are about the same as in the flow field. The pipe network is then a <u>finite difference approximation</u> of the flow field.



The pipe-network system of fluid flow can be modeled with a resistor-network system of electric flow.



Laplace's two-dimensional equation for flow through the aquifer is

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0;$$

and since head, h, is analogous to voltage, V, a similar steady-state flow equation for electricity intuitively follows:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0.$$

To convert values of voltage in the electric model into values of head, there must be some scale factor c_h such that $h = c_h V$. Furthermore, resistance values in the model must be such that

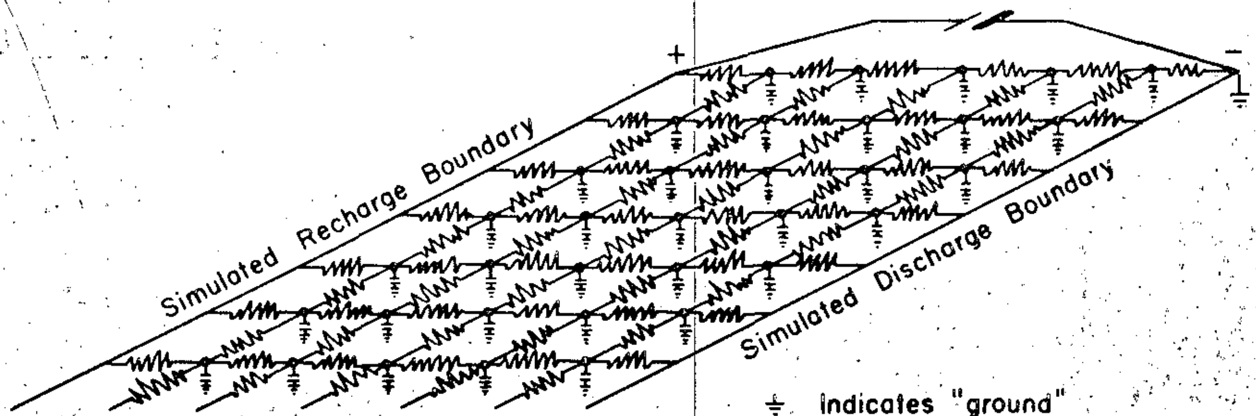
$$1/R = c_y Y$$

where c_y is a scale factor selected on the basis of current-voltage capabilities of the electric equipment used. In order to obtain values of head by using an electric-analog model it is necessary to measure voltage at each junction or node in the model, and multiply the voltage obtained by the scaling factor, c_h.

Now consider the case in which a ground-water system is described by the diffusion, or non-steady state, equation,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t}.$$

In this case, a change in storage occurs in the aquifer, and it is specified that rates of inflow differ from the rates of outflow. An electric-analog model may be constructed similar to the steady state system except that in addition there is a provision for storage at each node, a capacitor.



The analogous electric non-steady state equation is

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = RC\frac{\partial V}{\partial t},$$

where

$$C = c_s AS,$$

and

c_s is a scale factor,

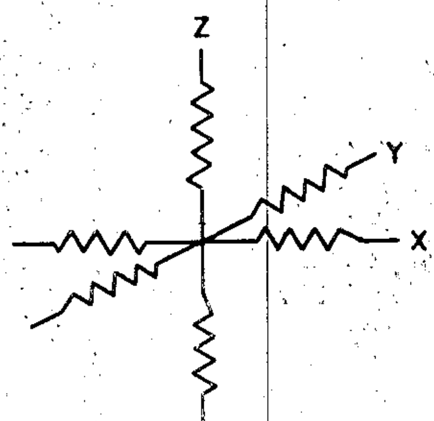
A is land surface area represented by a node,

S is aquifer coefficient of storage.

Again $h = c_h V$, however, since this is a transient system both head and its analog, voltage (potential) in any place in the system will be continuously changing. Hence a measurement of head or voltage obtained has meaning only if it has a time reference, and therefore in the transient case a provision for the graphic recording of voltage with time is necessary.

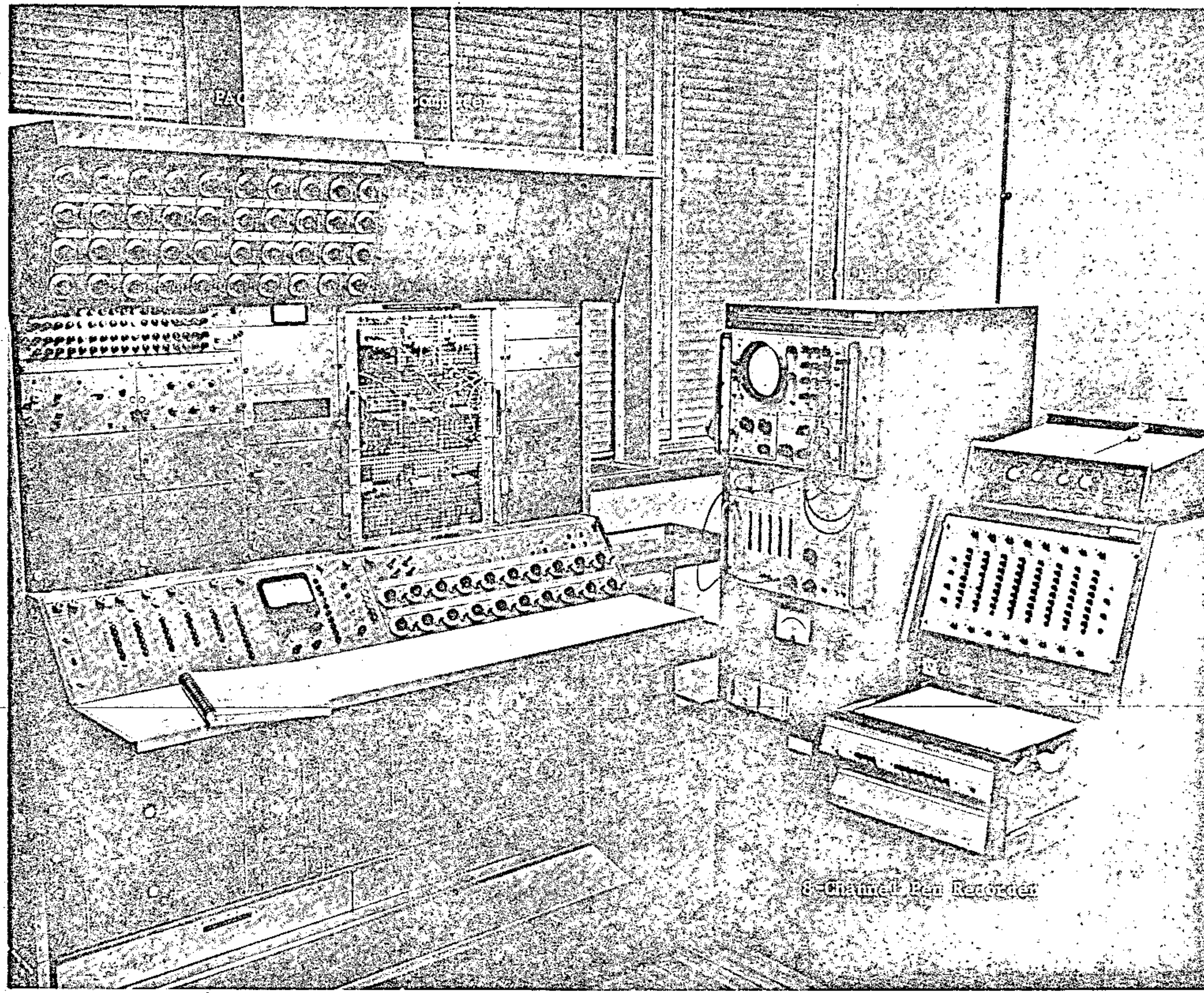
A non-steady flow condition in an aquifer is usually the result of pumpage from wells. Pumpage can be simulated in an electric-analog model by extracting current from the model at the appropriate nodes. The variations in rates of extraction of current corresponding to time-changes in pumping rates are accomplished by the application of current-generating devices.

Although electric-analog models described have been for two-dimensional flow, resistance-capacitance networks also can be constructed to simulate three-dimensional flow. The following diagram illustrates connections of resistors at a node in a three-dimensional steady-state flow field.



Electric-analog models are advantageous in the study of flow problems involving non-uniform permeability and irregularly-shaped boundaries.

The electric-analog models discussed have been the passive-element type because no elements in the models generated electric energy. If the contrary were true, as would be the case if electronic amplifiers were incorporated in a model, then the model would be an active-element type. Active elements are used in general purpose analog computers (electronic differential analyzers) which are discussed in the next chapter.



Electronics Associates Incorporated
Model PACE 221-R Analog Computer and Recording Equipment

Chapter VI

ANALOG COMPUTERS
(Electronic Differential Analyzers)

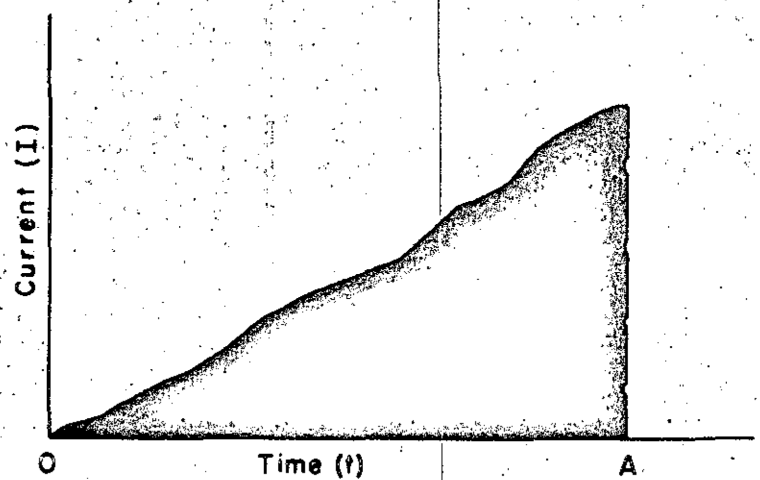
Analog computers are general purpose devices which employ laws of physics for performing mathematical operations. An example familiar to many is the slide rule. In this chapter an analog computer means specifically the electronic differential analyzer, so-named because of its application in solving ordinary differential equations. Analog computers are similiar in construction to the electric-analog models discussed in the previous chapter. However, an electric-analog model is constructed for a specific problem and is not capable of handling a wide variety of problems.

The analog computer has the same elements as the resistance-capacitance analog model and many more. However in the analog computer, components are not permanently connected to one another, but are wired into a central patch board unit where they are easily connected together in any desired combination.

Although an analog computer can be operated as a direct electric-analog model (passive element), it generally uses amplifiers (active elements) and a variety of other electronic devices, some of which are for adding and multiplying voltages. Mathematical integration problems (differential equations) can be solved by using capacitors in accordance with the following law:

$$V = \frac{1}{C}\int_{0}^{A} I dt.$$

This means that if there is a variable current flow into a capacitor which might graph as follows:



then the voltage on the capacitor at time, t = A, would be the area shaded under the curve. Hence, integration, a calculus procedure, is a method of obtaining the area under a curve. Integration is a mathematical process of summation but the process is physically simulated when electric current flows into a capacitor. The principal utility of this process is to obtain the solution of ordinary differential equations, which is done by integration. Differing slightly from the partial differential equation, an ordinary differential equation is one in which there are only two variables; in an analog computer these are voltage and time.

One of the principal differences between the analog computers and the analog models discussed is that computers use electronic amplifiers which compensate for electric energy losses. Hence electric energy can be generated in the system, which classifies an analog computer as an active element analog. Conversely the electric-analog model discussed in the previous chapter was a passive element system, because no electrical energy was generated in the system.

An analog computer has one or more graphic recording devices which can be connected to any part of the system to obtain voltage variations with respect to time.

In order to solve a partial differential equation with an analog computer, the equation must be expressed as an ordinary differential equation. For example, flow in an aquifer can be expressed as a system of non-steady state equations in which the Laplacian portion of each equation (the left side) is set in a fixed finite-difference form. The only other partial derivative remaining in each equation is $\partial h/\partial t$. Each equation is then an ordinary differential equation, which can be solved by the computer. An example of solution is presented in the following chapter.

Chapter VII

EXAMPLES OF SOLUTIONS OF HYDROLOGIC PROBLEMS WITH THE USE OF DIGITAL AND ANALOG TECHNIQUES

This chapter presents selected cases from the literature in order to show how computers and models have been applied to hydrologic problems, resulting in a considerable savings in man-hours of work which otherwise would have been required had these problems been approached through manual computational methods.

Digital Computer Methods

Perhaps the most widely used equation in quantitative ground-water analysis is the non-steady state equation describing radial flow to a discharging well. This equation is commonly known as Theis' non-equilibrium formula:

$$s = 114.6\frac{Q}{T}\int_{u}^{\infty}\frac{e^{-x}}{x}dx,$$

where $u = 1.87\frac{r^2 S}{tT}$,

and r = distance from pumped well, in feet,

s = drawdown of water level, in feet,

t = time since pumping began, in days,

Q = pumping rate, in gallons per minute,

S = coefficient of storage, dimensionless,

T = coefficient of transmissibility, in gallons per day per foot,

x = dummy variable for integration.

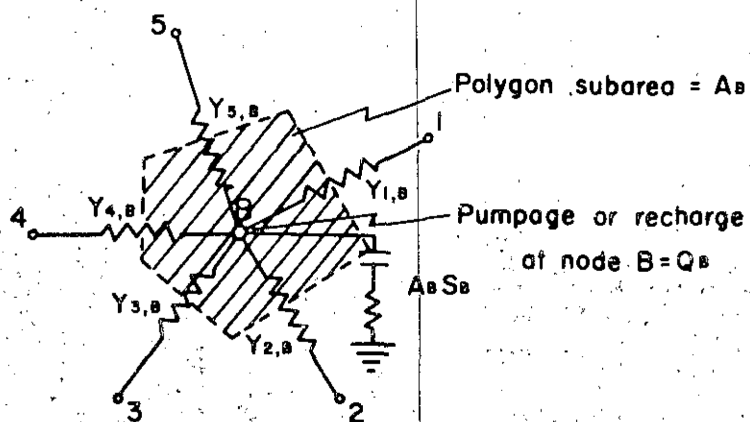
This formula determines the drawdown that would occur at any distance from a pumped well at any time after pumping began if values of coefficient of storage, coefficient of transmissibility, and pumping rate are known. Because the equation is tedious to solve, tables have been compiled to aid in its evaluation. Nevertheless, if very many values of drawdown are to be obtained the procedure is quite time consuming. A digital computer method of obtaining desired answers is used by this agency for aquifers with recharge boundaries. The program also provides for automatic machine plotting of results onto a graph. A similar program solves the equation for a system of arbitrarily spaced wells which may have complex pumping histories.

The California Department of Water Resources used a digital computer in the Los Angeles Coastal Plain area to analyze the effects on water levels as a result of possible future conditions of pumping and artificial recharge. Prior to this analysis the aquifer characteristics had been obtained by synthesis using an analog computer. In using the digital method for analysis the aquifer was treated as a flow system which satisfies the equation of continuity. For computational purposes, the aquifer was hypothetically modelled as an electric network problem. The digital computer solved the problem for water level at each node by a system of simultaneous differential equations expressed in finite difference form. The basic equation at each representative node for a given time increment Δt is

SUBSURFACE INFLOW - SUBSURFACE OUTFLOW + RECHARGE - EXTRACTIONS - CHANGE IN STORAGE = 0.

The corresponding finite difference equation and explanatory diagram are shown below.

$$\sum_{i=1}^{5}\left(h_i^{j+1}-h_B^{j+1}\right) Y_{iB} - Q_B^{j+1} = \frac{A_B S_B}{\Delta t}\left(h_B^{j+1}-h_i^{j}\right)$$



The superscripts, j and j+1, denote the ordered time increment applicable to the variable superscripted.

The water levels computed (h_i^{j+1}) are only for the end of a step or increment of time in the complete pumping history. The system of simultaneous equations must be solved for each successive time increment, beginning with the first ($j=1$). Computed future water levels are printed out or punched on cards and are later used for economic analysis.

One advantage of the digital computer method is that computed water-level data can be presented on punched cards or magnetic tape and used as input data in other digital programs. Another advantage is that a solution for a particular analysis problem can be easily obtained on short notice.

Digital computers are also useful in analysis of surface-water hydrologic data, particularly in the predictions of reservoir performance. Suppose that there are several possible sites for a reservoir, and it is desired to determine which location would be best from the standpoint of yield and storage characteristics. It is of particular interest to know how much water can be expected from the reservoir during a drought.

A fundamental equation of continuity applies to surface-water reservoirs:

INITIAL STORAGE + INFLOW - EVAPORATION - USE -

SPILLS = FINAL STORAGE.

The equation describes an inventory and can be balanced by a digital computer on a daily or monthly basis depending on availability of records. If recorded data from the worst drought are used, then a "firm yield" from the proposed reservoir can be predicted. Of particular appreciation is the immense saving in man-hours of tedious computational work which would otherwise have to be performed by engineers.

Analog Model Methods

Use of direct electric-analog models preceded the use of analog computers by many years. For example a resistance-capacitor network for analyzing fluid flow through porous media was developed in the early 1940's, several years before electronic analog computers were operational and commercially available.

In modeling a ground-water system with an electric analog, the aquifer system is first partitioned or divided into polygonal subareas. The flow of water through subareas is simulated by the flow of electricity through resistors and the values of resistances are selected so they represent hydraulic conductances of the aquifer. Capacitors at junctions of resistors (nodes) simulate storage properties of the aquifer. Values of coefficient of storage and hydraulic conductance are obtained from pumping-test data.

After the model is constructed it is tested to see if response of the model conforms to the historic response of the aquifer. To do this, pumpage history is simulated by a varying electric current which is injected into or extracted from the model at appropriate nodes. Voltages at the nodes are graphically recorded. The voltages, of course, simulate water levels in the aquifer. If the voltage graphs do not conform to the hydrographs obtained from field measurements of water levels the aquifer model is modified. That is, values of resistance and capacitance in various parts of the model are changed in order that the system may conform more closely to the historical records, assuming pumpage data are correct. This is the synthesis process. When the synthesis is carried out to where there is a satisfactory agreement in response of model and prototype, realistic values of hydraulic conductance and storage properties are, by application of the appropriate scaling factors, obtainable from the values of resistances and capacitances in the model.

It should be noted that aquifer coefficients obtained by model synthesis may differ from those derived from pumping tests. Coefficients obtained from pumping tests reflect aquifer properties only in the immediate vicinity of the pumped well and for a relatively short time. Therefore, aquifer parameters obtained from pumping tests are generally modeled only as preliminary approximations, and are later adjusted by trial and modification procedures until the model's response simulates with sufficient accuracy historic water-level data.

Of particular interest in the use of analog models in ground-water problems is a study recently completed in the vicinity of Houston. In the Houston area most development is from two sand aquifers separated from each other by a rather thick, sandy clay layer. There is significant hydraulic connection between these aquifers, and there is considerable interbedded clay in the lower aquifer. With such a complex aquifer system compounded by withdrawals from so many large-capacity wells the estimation of future water-level declines in the Houston area would be practically impossible with manual computational methods. Coefficients of transmissibility and storage of the aquifer were obtained from field pumping tests and were simulated as preliminary approximations in a resistance-capacitance network. Vertical permeability of the clay layer between the aquifers was estimated. After several trials and modifications, the model was finally synthesized so that its response conformed reasonably with historic water levels.

The analysis phase of the Houston problem consisted of imposing simulated anticipated future pumping conditions and recording the expected water-level declines.

The model used for the Houston problem utilized a very short time scale, on the order of one millionth of a second of model time to one year of actual time. Although short time scaling is sometimes desirable in order to take advantage of low voltages and small inexpensive capacitors, expensive equipment is required for simulation of pumpage. In the Houston problem pumpage was simulated by several square-wave generators connected such that their combined output graphed as a pumpage histogram. The simulated water-level hydrographs at each node were presented on the screen of an oscilloscope. In order to have a record of the simulated water-level hydrograph it was necessary to photograph the pattern displayed on the oscilloscope screen.

Electric-analog models have been used to some extent in surface-water work, particularly in flood control and tidal studies, and are considerably more complicated than those used in ground-water problems. Applied research in this field has been conducted at the hydraulics laboratory of the University of California at Berkeley.

A surface-water problem in California investigated with an electric-analog method involves two rivers, the San Joaquin and the Sacramento. The Sacramento River drains the northern part of the State, and the San Joaquin River drains the southern part. The confluence of the rivers, about 60 miles east of San Francisco, is a poorly drained area characterized by an intricate pattern of interlaced waterways, called the Delta. From the Delta, which is approximately at sea level, drainage is westerly through a series of small bays into the San Francisco Bay, thence through the Golden Gate into the Pacific Ocean.

Because the Delta is situated so close to sea level and near the ocean, surface-water supplies are subjected to contamination by salt-water intrusion owing to tides. Furthermore because the San Joaquin River drains the relatively arid and agricultural southern part of the state, it transmits poor quality water to the Delta in summer months.

Oceanic tides are periodic pulsations, and in the river and bay systems from the Delta to the Pacific Ocean these tidal pulsations generate complicated patterns of wave reflections, some of which may be mutally reinforcing.

It is desired to make some changes in the channel system which will consist of a barrier or an obstruction to the intrusion of sea water into the Delta, and the changes involved will be rather costly and extensive. Therefore an analog model of the channel system would appear to be desirable in order to study tidal effects on the channel system under proposed modification. After a variety of proposed changes are studied, optimum modification plan can be selected. Furthermore it is also desirable to model the channel in order to study behavior of floods from the San Joaquin and Sacramento Rivers so that they can be effectively controlled.

The construction and application of an electric-analog model of a river system presents many technical difficulties, because not only must channel characteristics be modelled (and these characteristics change with depth of water in the channel) but also appropriate time-delay devices must be built into the system, and input equipment must be designed to simulate flood characteristics. Similar difficulties are encountered in tide simulation.

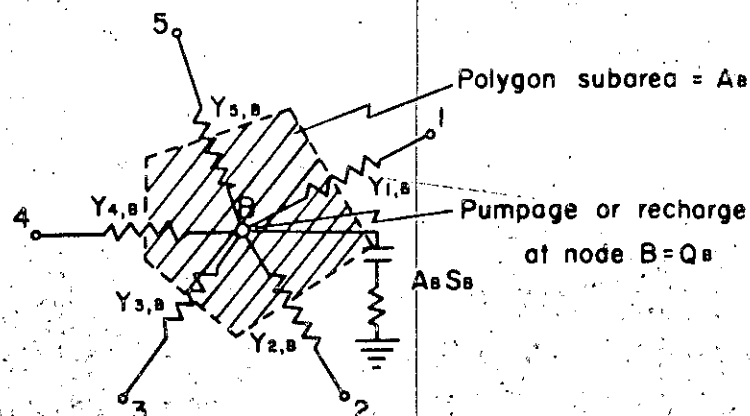
Analog Computer Methods

The California Department of Water Resources used an analog computer (electronic differential analyzer) for aquifer synthesis in the Los Angeles Coastal Plain. The analog computer was used as an active element device and the aquifer was not simulated directly, hence the correspondence of properties between prototype and model are not apparent.

Before describing the application of the analog computer to the solution of the ground-water problem in the Los Angeles area, it is desirable at this time to point out some of the attractive features of the analog computer compared to an analog model. First, analog computer components are connected to a central patch board, which facilitates hook-up operations. Moreover, the different elements used to simulate storative and transmissive characteristics of the aquifer are variable, hence adjustments are easily performed. Adjustments are not easily made in a direct-analog model if fixed resistors and capacitors are used to simulate the aquifer's characteristics, in which case changes can be made only by substitution of elements. Another advantage of the analog computer is the large model-time scale used. In contrast to the time scale of one micro-second per year in the analog model of the Houston problem a time scale of approximately one second per year was used in the Los Angeles problem on the analog computer. The longer time scale allows the output hydrographs to be recorded graphically by a pen plotter rather than an oscilloscope. Pen plotters are generally more accurate than oscilloscopes of comparable quality. By using a multi-channel plotter on an analog computer, time-voltage graphs at several nodes can be obtained simultaneously.

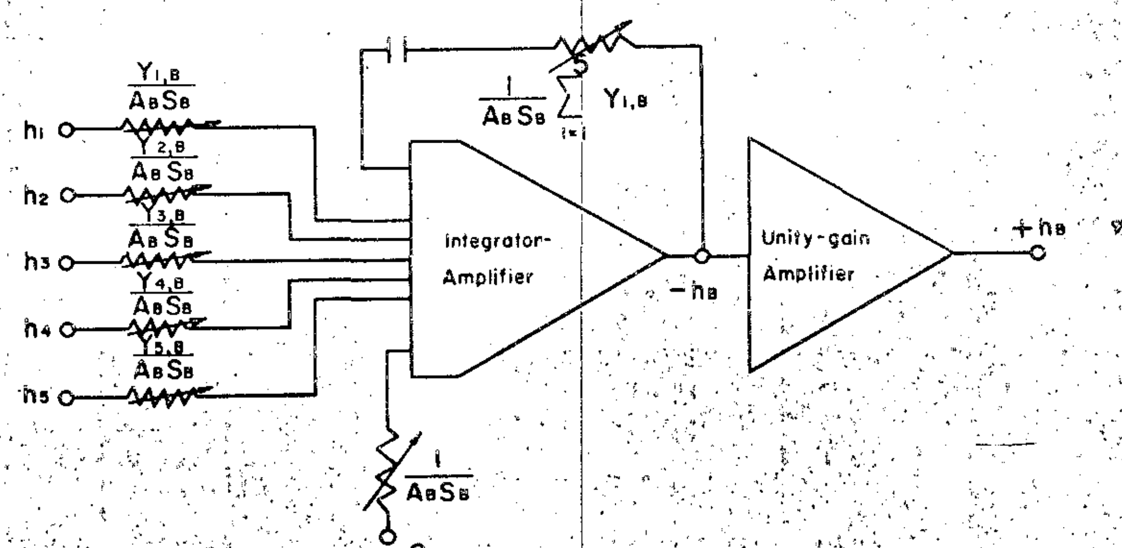
In the Los Angeles Coastal Plain problem the aquifer was modelled by three PACE 231-R analog computers, operating simultaneously. In order to utilize components most effectively, and model as accurately as possible the irregular aquifer boundaries and irregularly spaced centers of pumping and artifical re-charge, an asymetric pattern of nodes rather than a regular square pattern was used. The differential equation of flow at each node and schemetic diagram for the connection of components at a node in the computer are illustrated on the following page.

$$\left\{\sum_{i=1}^{5}(h_i - h_B)Y_{i,B}\right\} - Q_B = A_B S_B \frac{\partial h_B}{\partial t}$$



$$\frac{dh_B}{dt} = \frac{1}{A_B S_B}\left\{\sum_{i=1}^{5}(h_i - h_B)Y_{i,B}\right\} - \frac{Q_B}{A_B S_B}$$

$$h_B = \int_{t_1}^{t_2}\left\{\sum_{i=1}^{5}\frac{Y_{i,B}}{A_B S_B}h_i - \frac{Q_B}{A_B S_B} - h_B\sum_{i=1}^{5}\frac{Y_{i,B}}{A_B S_B}\right\}dt$$



The actual circuit employed was modified slightly from that shown to conserve time in making adjustments. However, the schematic diagram shown is basically correct and could have been used. The inputs into integrators are the subsurface flows and the surface recharge and extraction. They are integrated by accumulation on a condenser. The integrator output is a voltage which varies with time (simulated water-level variations with time). The integrator output is fed into a unity-gain amplifier to reverse polarity (integrators and amplifers are constructed such that the polarity of the output signal is opposite of the input signal).

Mostly for comparative purposes, the California Department of Water Resources used the analog computer in the analysis of the aquifer in the Los Angeles Coastal Plain. Although the two procedures yielded practically identical results the digital computer appeared to be a more useful device for aquifer analysis, particularly because of convenience and because of savings in time and cost.

It is therefore of interest to note that in planning for the optimum development and management of surface- and ground-water supplies in the Los Angeles area both analog and digital computers were used. The analog computer was used principally to define or sythesize the aquifer. The digital computer was used thereafter to analyze the effects on water levels that would occur from various proposed schedules of pumping and artificial recharge.

However, this is but a part of the water resources problem in this area. At the present time, much of the water used in the Los Angeles area is obtained from distant surface-water sources which are developed in conjunction with local ground-water supplies. Moreover, it is anticipated that additional water will soon be imported from the Feather River Project, originating in the northern part of the state. Therefore, the most optimum plan of development of available surface- and ground-water resources should be analyzed. Analysis of such information requires the handling of a large volume of statistical data and data concerned with costs, available storage, future pumping demands, and so on. Processing of such volumes of information and choosing the optimum plan of water development suggest the use of a digital computer. Many interrelated programs are therefore utilized in order to achieve an efficient plan of water management.

CONCLUSION

A decision to solve a hydrologic problem through one or more of the techniques of digital computer, analog computer or analog model should be based primarily on whether a necessity exists for obtaining the solution sought. If the need does exist, then available pertinent data should be collected and carefully scrutinized to determine (1) the probability of obtaining reasonable results and (2) which technique is most practical.

One way to decide on the technique to be used is to personally interrogate various experienced individuals. Perusal of published literature is recommended, but it must be remembered that technical papers often contain some personal vanity. That is,many writers are usually inclined to report favorably on their successes, dismissing the difficulties encountered in their efforts. Reports of failures seldom appear in the technical journals.

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