# Volumetric Survey of LAKE PAT CLEBURNE

July 2008 Survey



Prepared by:

The Texas Water Development Board

October 2008

# Texas Water Development Board

J. Kevin Ward, Executive Administrator

Texas Water Development Board

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# Prepared for:

# **City of Cleburne**

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# **Executive Summary**

In April of 2008, the Texas Water Development Board (TWDB) entered into agreement with the City of Cleburne, Texas, for the purpose of performing a volumetric survey of Lake Pat Cleburne. Cleburne Dam and Lake Pat Cleburne are located on Nolan Creek in the Brazos River basin near Cleburne, Texas, in Johnson County. TWDB conducted the Lake Pat Cleburne survey on July 8<sup>th</sup>-10<sup>th</sup>, 2008 while the average water surface elevation of the lake was 732.29 feet above mean sea level (NGVD 29). The conservation pool elevation for the lake is 733.5 feet above mean sea level (NGDV 29).

The results of the TWDB 2008 Volumetric Survey indicate Lake Pat Cleburne has a total reservoir capacity of 26,008 acre-feet and encompasses 1,568 acres at conservation pool elevation (733.5 feet above mean sea level, NGVD 29). Lake Pat Cleburne capacity (at conservation pool elevation) was previously estimated at 25,730 acre-feet in 1998 (by TWDB¹) and 25,560 acre-feet in 1958 (TWDB & Hunter Associates²). Due to differences in the methodologies used in calculating areas and capacities from this and previous Lake Pat Cleburne surveys, comparison of these values is not recommended. The TWDB considers the 2008 survey to be a significant improvement over previous methods and recommends that a similar methodology be used to resurvey Lake Pat Cleburne in approximately 10 years or after a major flood event.

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**Note:** References to brand names throughout this report do not imply endorsement by the Texas Water Development Board

# **Lake Pat Cleburne General Information**

Cleburne Dam and Lake Pat Cleburne are located on Nolan Creek, a tributary of the Brazos River, near Cleburne, Texas, in Johnson County<sup>2</sup> (Figure 1). The City of Cleburne, Texas owns and operates Lake Pat Cleburne. Construction of Cleburne Dam began on August 9, 1963, with deliberate impoundment beginning on August 4, 1964.<sup>2</sup> Lake Pat Cleburne serves as a municipal, industrial, and agricultural water supply for the city, also providing flood control and recreational opportunities for the local community.<sup>1</sup> Additional pertinent data about Cleburne Dam can be found in Table 1.

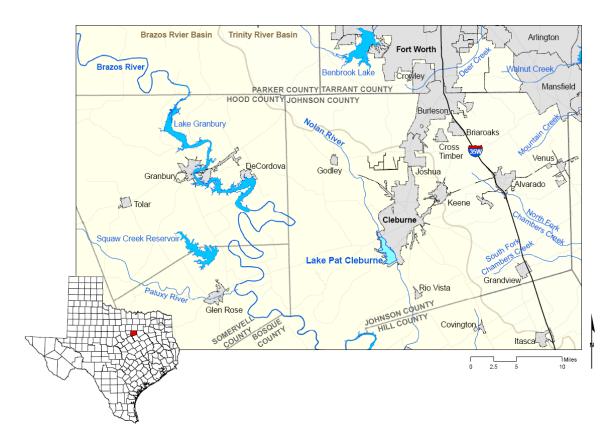


Figure 1 - Location Map: Lake Pat Cleburne

Table 1. Pertinent Data for Cleburne Dam and Lake Pat Cleburne

Owner

City of Cleburne

Engineer (Design)

**Hunter Associates** 

**Location of Dam** 

On Nolan River in Johnson County, 4 miles south of Cleburne, TX

Drainage Area

100 square miles

Dam

Type Earthfill

Length

(Does not include spillways) 4,950 feet Maximum height 78 feet Top width 25 feet

Spillway (emergency)

Type Excavated channel

Control None

Elevation 744.0 feet above mean sea level

Crest length 500 feet

Spillway (service)

Type Concrete ogee

Control None

Elevation 733.5 feet above mean sea level

Crest length 150 feet

**Outlet Works** 

Control

Type Tower and 30-inch diameter steel pipe inside of 36-inch

concrete pipe 2 sluice gates

Invert elevation 690.0 feet above mean sea level

Reservoir Data (Based on TWDB 2008 Volumetric Survey)

Feature	Elevation	Capacity Area	
	(feet above mean sea level)	(Acre-feet)	(Acres)
Top of Dam	753.0	N/A	N/A
Top of design flood pool	752.3	N/A	N/A
Emergency spillway crest	744.0	N/A	N/A
Service spillway crest	733.5	26,008	1,568
Invert of upper sluice gate	722.0	11,050	969
Invert of lower sluice gate	690.0	0	0
Usable conservation storag	re -	26,008	

#### **Water Rights**

The water rights for Lake Pat Cleburne are owned by the City of Cleburne through Certificate of Adjudication No. 12-4106 and its amendments. A brief summary of each follows. The complete certificates are on file in the Records Department of the Texas Commission on Environmental Quality.

#### Certificate of Adjudication No. 12-4106

Priority Dates: August 6, 1962 & March 29, 1976.

Authorizes the City of Cleburne to maintain an existing dam and reservoir, Lake Pat Cleburne, on the Nolan River and impound therein a maximum of 25,600 acre-feet of water. The City of Cleburne is authorized to divert and use a maximum of 5,760 acre-feet of water per year for municipal purposes. The City is also authorized to divert and use a maximum of 240 acre-feet of water per year to irrigate up to 80 acres of land that lies northeast of the reservoir. All water that is diverted and not consumed is to be returned to Buffalo Creek, a tributary of the Nolan River.

#### Amendment to Certificate of Adjudication No. 12-4106A

Granted: October 11, 1996

Authorizes the City of Cleburne to divert and use the 5,760 acre-feet of water originally authorized for municipal purposes, for municipal and industrial purposes. This amendment also requires the City of Cleburne to implement a water conservation plan and requires all industrial users to have a method of measuring the quantity of water diverted for that purpose.

#### Amendment to Certificate of Adjudication No. 12-4106B

Granted: January 15, 2002

Authorizes the City of Cleburne to use the bed and banks of Lake Pat Cleburne to deliver water purchased from the Brazos River Authority. The City is then authorized to divert and use 5,300 acre-feet of contracted water, delivered by pipeline from Lake Aquilla, for municipal purposes (pursuant to the Water Sale Contract between the City of Cleburne and the Brazos River Authority, Certificate of Adjudication No. 12-5158). The amendment also authorizes the City of Cleburne to divert, from currently authorized diversion points, and use, an additional 4,700 acre-feet of water per year of Lake Whitney contract water for municipal purposes (pursuant to a Water Sale Contract between the City of Cleburne and the Brazos River Authority, Certificate of Adjudication No. 12-5157). This amendment also requires the City to implement a water conservation plan, and requires every wholesale customer entered into contract with to implement a water conservation plan.

## Amendment to Certificate of Adjudication No. 12-4106C

Granted: November 30, 2005

Authorizes the City of Cleburne to divert and reuse 8,400 acre-feet of existing and future return flows originating from two wastewater treatment plants for agricultural, industrial, and municipal purposes within the City's service area in Johnson County. The City is authorized to use the bed and banks of West Buffalo Creek, Buffalo Creek, and the Nolan River to convey return flows to the downstream diversion point. The priority date for diverting 8,400 acre-feet of water is August 20, 2004. The first 3,684 acre-feet of diversion is subject to call by senior water right holders, and is restricted by streamflow rates measured either directly downstream of the diversion point or at USGS Gage #08092000 Nolan River and Blum. The remaining 4,716 acre-feet of the authorized 8,400 acre-feet is not subject to call by senior water right holders nor instream flow restrictions.

#### Volumetric Survey of Lake Pat Cleburne

#### Introduction

The Hydrographic Survey Program of the Texas Water Development Board (TWDB) was authorized by the state legislature in 1991. The Texas Water Code authorizes TWDB to perform surveys to determine reservoir storage capacity, sedimentation levels, rates of sedimentation, and projected water supply availability.

In April of 2008, the TWDB entered into agreement<sup>3</sup> with the City of Cleburne, Texas, for the purpose of performing a volumetric survey of Lake Pat Cleburne. This report describes the methods used in conducting the volumetric survey, including data collection methods and data processing techniques. This report serves as the final contract deliverable from TWDB to the City of Cleburne, and contains as deliverables: (1) an elevation-area-capacity table of the lake acceptable to the Texas Commission on Environmental Quality, (2) a bottom contour map, and (3) a shaded relief plot of the lake bottom.

#### **Datum**

The vertical datum used during this survey is that used by the United States Geological Survey, for the reservoir elevation gauge USGS 08091900 Lk Pat Cleburne nr Cleburne, TX. The datum for this gauge is reported as National Geodetic Vertical Datum 1929 (NGVD 29) or mean sea level, thus elevations reported here are in feet above mean sea level. Volume and area calculations in this report are referenced to water levels provided by the USGS gauge. The horizontal datum used for this report is North American Datum of 1983 (NAD83) State Plane Texas North Central Zone (feet).

#### **TWDB Bathymetric Data Collection**

Bathymetric data collection occurred July 8<sup>th</sup> through July 10<sup>th</sup> 2008 while the daily average water surface elevation of the lake was 732.29 feet above mean sea level (NGVD 29). For data collection, TWDB used a Knudsen Engineering Ltd. single-frequency (200 kHz) depth sounder integrated with Differential Global Positioning System (DGPS) equipment. Data collection occurred while navigating along pre-planned range lines oriented perpendicular to the location of the original river channels and spaced approximately 500 feet apart. Data was also collected along some of the survey lines used during the 1998 Lake Pat Cleburne survey conducted by TWDB. For all data collection efforts, the depth sounder was calibrated daily by comparing depth readings recorded by the Knudsen echosounder to physical depth measurements made with a weighted tape and stadia rod. During the 2008 survey, team members collected over 20,200 data points over cross-sections totaling nearly 38 miles in length. Figure 2 shows where data points were collected during the TWDB 2008 survey.

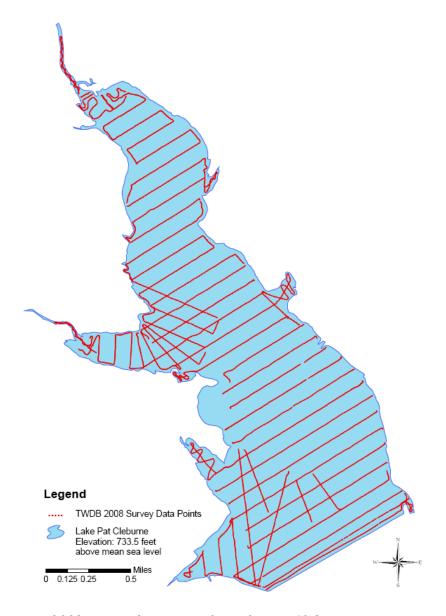


Figure 2 – TWDB 2008 survey data points for Lake Pat Cleburne

# **Data Processing**

#### **Model Boundaries**

The boundary of Lake Pat Cleburne was manually digitized from digital ortho quarter quadrangle (DOQQ) aerial photos<sup>5</sup> available from the Texas Natural Resources Information System (TNRIS).<sup>6</sup> The model boundary at conservation pool elevation (733.5.0 feet) was digitized from the Cleburne West NW, NE, SW, and SE DOQQs photographed on August 4, 2004 when the water surface elevation in Lake Pat Cleburne was 733.34 feet as measured by the United States Geological Survey (USGS) gauge at

Cleburne Dam. As the DOQQs used in digitizing the boundary are of 1-meter resolution, the physical boundary of Lake Pat Cleburne may be within  $\pm$  1 meter of the location derived from the manual delineation. For the purposes of this analysis it was assumed that the boundary of the lake at conservation pool elevation is closely represented by the August 4, 2004 DOQQ. Therefore for the purpose of calculating elevation-area-capacity tables, TWDB assigned the digitized boundary an elevation of 733.5 feet.

Additional lake boundary data were derived through digitization of DOQQs derived from aerial photographs taken on August 19, 2006. On this date, the water surface elevation in Lake Pat Cleburne averaged 730.31 feet according to the USGS gauge. This additional dataset was used as a supplement to the TWDB survey data in locations where the survey data alone was insufficient to properly represent the reservoir bathymetry.

#### Triangular Irregular Network (TIN) Model

Upon completion of the data collection effort, the raw data files collected by TWDB were edited using customized MATLAB processing scripts and the HydroEdit software package. Specifically, HydroEdit applies a median filter to the raw survey data and removes individual data anomalies or points with incorrect GPS coordinates. HydroEdit also uses the water surface elevations at the times of each sounding to convert sounding depths to corresponding bathymetric elevations<sup>7</sup>. The MATLAB processing scripts are then used to visually inspect each of the filtered cross-sections to indentify and rectify any series of data anomalies that were not edited using the HydroEdit filters. For processing outside of MATLAB and HydroEdit, the sounding coordinates (X,Y,Z) are exported as a MASS points file. TWDB also created a MASS points file of interpolated data located in-between surveyed cross sections. This points file is described in the section entitled "Self-Similar Interpolation."

To create a surface representation of the Lake Pat Cleburne bathymetry, the 3D Analyst Extension of ArcGIS (ESRI, Inc.) is used. This extension creates a triangulated irregular network (TIN) model of the bathymetry, where each MASS point and boundary node becomes the vertex of a triangular portion of the reservoir bottom surface. From the TIN model, reservoir capacities and areas are calculated at one-tenth of a foot (0.1 foot) intervals, from elevation 692.0 feet to elevation 733.5 feet.

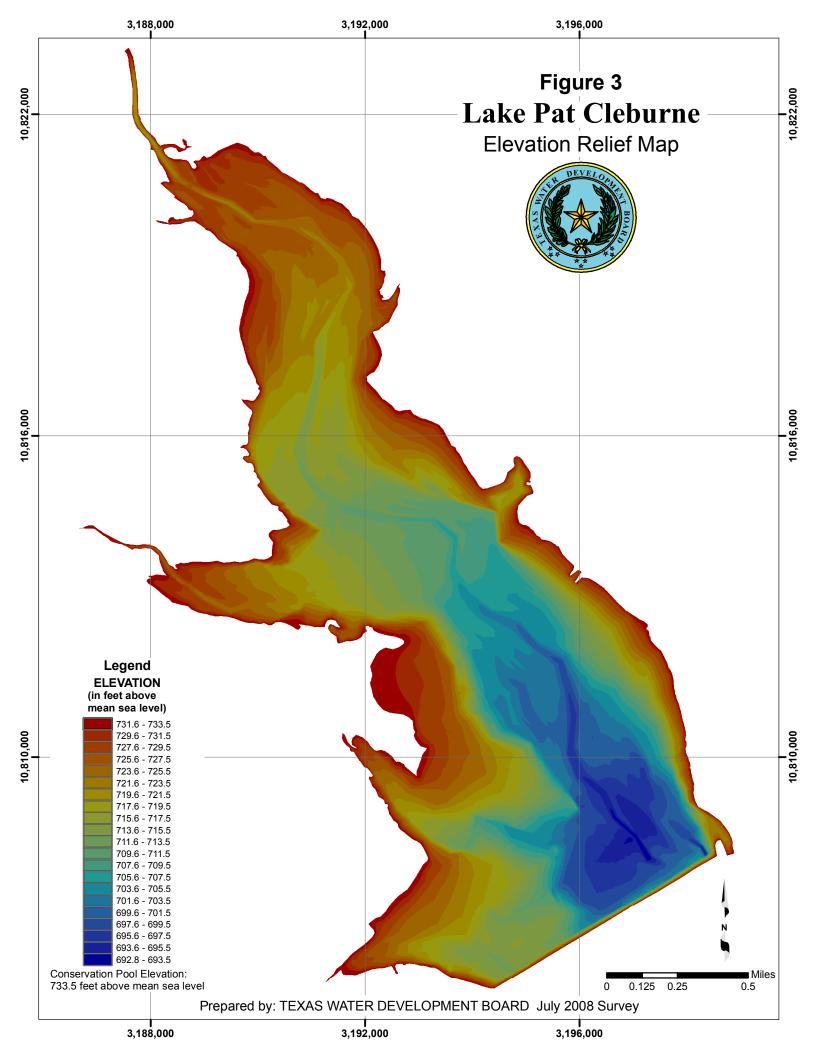
The Elevation-Capacity and Elevation-Area Tables, updated for 2008, are presented in Appendices A and B, respectively. Tables are provided with elevations referenced to the NGVD 29 datum. An Elevation-Area-Capacity graph is presented in Appendix C.

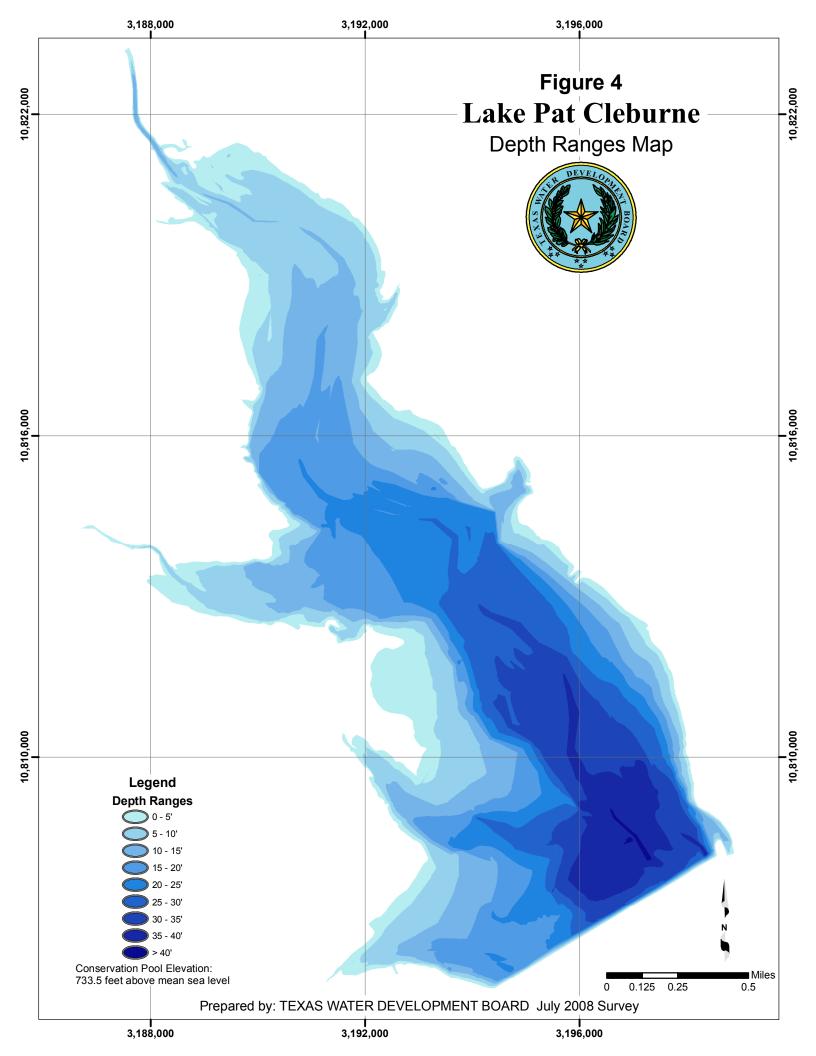
The Lake Pat Cleburne TIN model was interpolated and averaged using a cell size of 1 foot by 1 foot and converted to a raster. The raster was used to produce an Elevation Relief Map representing the topography of the reservoir bottom (Figure 3), a map showing shaded depth ranges for Lake Pat Cleburne (Figure 4), and a 2-foot contour map (Figure 5 - attached). The reservoir extent depicted in these figures is that corresponding to the conservation pool elevation (733.5 feet).

#### **Self-Similar Interpolation**

A limitation of the Delaunay method for triangulation used by ESRI products for creating TIN models results in artificially-curved contour lines extending into the reservoir where the reservoir walls are steep and the reservoir is relatively narrow. These curved contours are likely a poor representation of the true reservoir bathymetry in these areas. Also, if the surveyed cross sections are not perpendicular to the centerline of the submerged river channel (the location of which is often unknown until after the survey), then the TIN model is not likely to well-represent the true channel bathymetry.

To ameliorate these problems, a Self-Similar Interpolation routine (developed by TWDB) was used to interpolate the bathymetry between many 500 foot-spaced survey lines. The Self-Similar Interpolation technique effectively increases the density of points input into the TIN model, and directs the TIN interpolation to better represent the reservoir topography. In the case of Lake Pat Cleburne, the application of Self-Similar Interpolation helped represent the lake morphology near the banks and improved the representation of the submerged river channel (Figure 6). In areas where obvious geomorphic features indicate a high-probability of cross-section shape changes (e.g. incoming tributaries, significant widening/narrowing of channel, etc.), the assumptions used in applying the Self-Similar Interpolation technique are not likely to be valid; therefore, self-similar interpolation was not used in areas of Lake Pat Cleburne where a high probability of change between cross-sections exists.





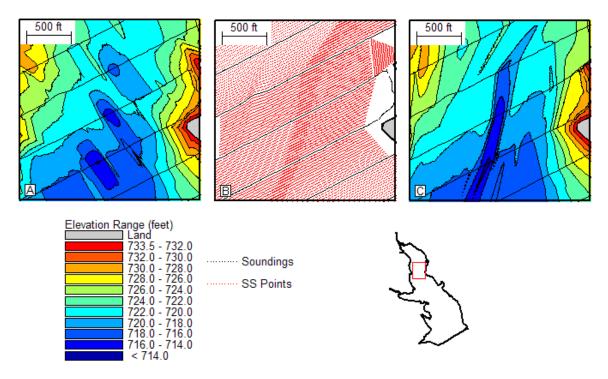


Figure 6 - Application of the Self-Similar Interpolation technique to Lake Pat Cleburne sounding data – A) bathymetric contours without interpolated points, B) Sounding points (black) and interpolated points (red) with reservoir boundary shown at elevation 733.5 feet (black), C) bathymetric contours with the interpolated points. Note: In 6A the deep channel indicated by the surveyed cross sections is not continuous within the areas inbetween the cross sections. This is an artifact of the TIN generation routine when data points are not collected perpendicular to the channel axis. Inclusion of the interpolated points (6C) corrects this and smoothes the bathymetric contours.

# **Volumetric Survey Results**

The results of the TWDB 2008 Volumetric Survey indicate Lake Pat
Cleburne has a total reservoir capacity of 26,008 acre-feet and encompasses 1,568
acres at conservation pool elevation (733.5 feet above mean sea level, NGVD 29).

Lake Pat Cleburne capacity (at conservation pool elevation) was previously estimated at
25,730 acre-feet in 1998 (by TWDB¹) and 25,560 acre-feet in 1958 (TWDB & Hunter
Associates²). Due to differences in the methodologies used in calculating areas and
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is not recommended.<sup>9</sup> The TWDB considers the 2008 survey to be a significant
improvement over previous methods and recommends that a similar methodology be used
to resurvey Lake Pat Cleburne in approximately 10 years or after a major flood event.

# **TWDB Contact Information**

More information about the Hydrographic Survey Program can be found at:

http://www.twdb.state.tx.us/assistance/lakesurveys/volumetricindex.asp

Any questions regarding the TWDB Hydrographic Survey Program may be addressed to:

Barney Austin, Ph.D., P.E. Director of the Surface Water Resources Division

Phone: (512) 463-8856

Email: Barney.Austin@twdb.state.tx.us

Or

Jason Kemp

Team Leader, TWDB Hydrographic Survey Program

Phone: (512) 463-2465

Email: Jason.Kemp@twdb.state.tx.us

# References

- 1. Texas Water Development Board, 1998, "Volumetric Survey of Lake Pat Cleburne"
- 2. Texas Water Development Board, Report 126, Engineering Data on Dams and Reservoirs in Texas, Part II, November 1973.
- 3. Texas Water Development Board Contract No. 0804800813 with the City of Cleburne, executed on 4/15/2008.
- 4. United States Geological Survey, Site 08091900 Lk Pat Cleburne nr Cleburne, TX http://waterdata.usgs.gov/tx/nwis/uv/?site\_no=08091900 viewed on 18 July 2008.
- 5. U.S Department of Agriculture, Farm Service Agency, Aerial Photography Field Office, National Agriculture Imagery Program, http://www.apfo.usda.gov/NAIP.html viewed 10 February 2006.
- 6. Texas Natural Resources Information System, TNRIS Home, http://www.tnris.state.tx.us/viewed 18 July 2008.
- 7. Furnans, Jordan. Texas Water Development Board. 2006. "HydroEdit User's Manual."
- 8. ESRI, Environmental Systems Research Institute. 1995. ARC/INFO Surface Modeling and Display, TIN Users Guide.
- 9. United States Department of Agriculture, Natural Resource Conservation Service, National Engineering Handbook, Section 3, Sedimentation, Chapter 7, Field Investigations and Surveys, December 1983.

#### Appendix A

# Lake Pat Cleburne RESERVOIR CAPACITY TABLE

# TEXAS WATER DEVELOPMENT BOARD CAPACITY IN ACRE-FEET

JULY 2008 SURVEY
Conservation Pool Elevation 733.5 Feet NGVD 29

ELEVATION INCREMENT IS ONE TENTH FOOT

ELEVATION										
in Feet	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
692	0	0	0	0	0	0	0	0	0	0
693	0	0	0	0	1	1	1	1	2	2
694	2	3	3	4	4	5	6	6	7	8
695	10	11	12	14	15	17	19	21	24	26
696	29	31	35	38	41	45	49	52	57	61
697	66	70	76	81	86	92	98	104	111	117
698	124	131	139	147	154	163	171	180	188	197
699	207	216	226	236	246	257	268	279	290	302
700	314	327	340	352	366	379	393	406	421	435
701	449	464	479	494	510	526	542	558	575	592
702	610	627	645	663	682	701	720	739	759	779
703	799	820	841	862	884	906	928	950	973	996
704	1,020	1,044	1,068	1,092	1,117	1,142	1,167	1,193	1,218	1,244
705	1,271	1,297	1,324	1,352	1,379	1,407	1,435	1,464	1,493	1,522
706	1,552	1,582	1,612	1,643	1,674	1,705	1,736	1,768	1,800	1,832
707	1,864	1,897	1,930	1,964	1,997	2,031	2,066	2,100	2,135	2,170
708	2,205	2,240	2,276	2,312	2,348	2,384	2,421	2,458	2,495	2,532
709	2,570	2,608	2,647	2,685	2,724	2,763	2,803	2,843	2,883	2,923
710	2,964	3,004	3,046	3,087	3,129	3,172	3,214	3,257	3,301	3,344
711	3,389	3,433	3,478	3,523	3,569	3,615	3,662	3,709	3,757	3,804
712	3,853	3,901	3,950	4,000	4,049	4,099	4,150	4,201	4,252	4,304
713	4,356	4,409	4,462	4,515	4,569	4,623	4,678	4,734	4,789	4,846
714	4,903	4,960	5,018	5,077	5,136	5,196	5,256	5,317	5,379	5,440
715	5,503	5,565	5,629	5,692	5,757	5,821	5,886	5,952	6,018	6,085
716	6,152	6,219	6,287	6,356	6,424	6,494	6,563	6,634	6,704	6,775
717	6,847	6,919	6,991	7,065	7,138	7,212	7,287	7,363	7,438	7,515
718	7,592	7,669	7,747	7,826	7,904	7,984	8,063	8,143	8,224	8,305
719	8,386	8,468	8,551	8,633	8,717	8,800	8,884	8,969	9,054	9,140
720	9,225	9,312	9,399	9,486	9,574	9,662	9,751	9,841	9,930	10,021
721	10,112	10,203	10,295	10,387	10,480	10,573	10,667	10,762	10,857	10,953
722	11,050	11,147	11,245	11,343	11,442	11,541	11,641	11,742	11,843	11,944
723	12,046	12,149	12,252	12,355	12,459	12,564	12,669	12,774	12,881	12,988
724	13,095	13,203	13,312	13,422	13,532	13,642	13,754	13,865	13,978	14,091
725	14,205	14,319	14,434	14,549	14,665	14,782	14,899	15,017	15,135	15,254
726	15,374	15,494	15,615	15,737	15,859	15,982	16,105	16,230	16,355	16,480
727	16,607	16,734	16,861	16,990	17,119	17,249	17,380	17,511	17,643	17,776
728	17,910	18,044	18,178	18,314	18,450	18,587	18,724	18,862	19,001	19,140
729	19,280	19,421	19,562	19,704	19,847	19,990	20,134	20,278	20,422	20,567
730	20,712	20,858	21,004	21,150	21,298	21,445	21,593	21,742	21,890	22,039
731	22,188	22,337	22,487	22,637	22,787	22,938	23,088	23,240	23,391	23,543
732	23,694	23,847	23,999	24,152	24,305	24,458	24,612	24,766	24,920	25,075
733	25,229	25,385	25,540	25,696	25,852	26,008				
·										

#### Appendix B

#### **Lake Pat Cleburne RESERVOIR AREA TABLE**

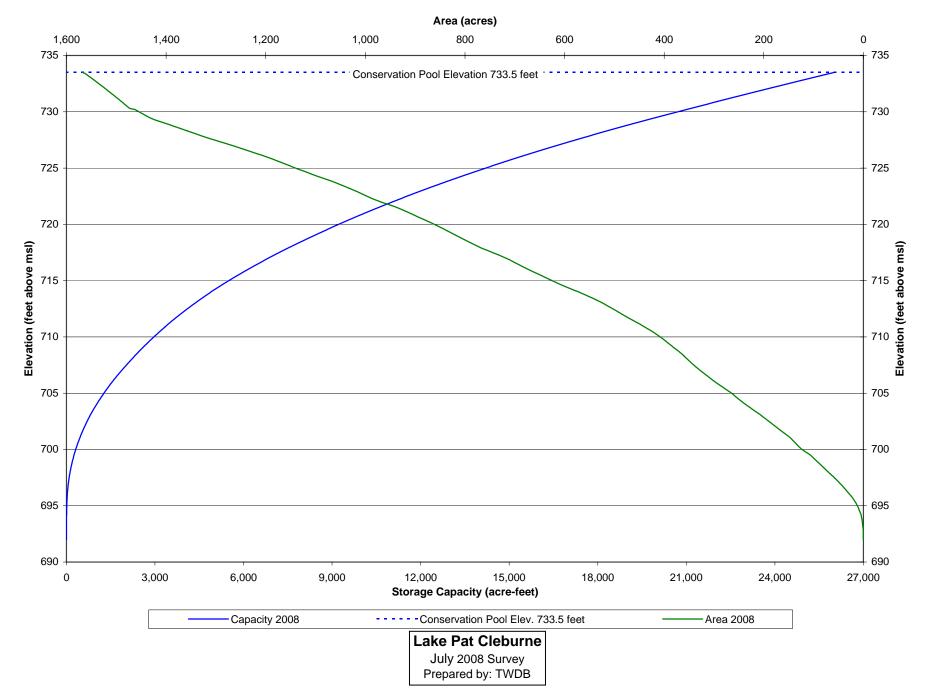
# TEXAS WATER DEVELOPMENT BOARD

AREA IN ACRES

JULY 2008 SURVEY Conservation Pool Elevation 733.5 Feet NGVD 29

ELEVATION INCREMENT IS ONE TENTH FOOT

In Feet	ELEVATION										
693         1         1         1         2         2         2         2         3         2         22         24         25         696         696         27         29         31         33         35         37         39         41         43         45         697         47         49         52         54         56         58         61         63         66         68         698         71         73         75         78         80         82         85         87         90         92         699         94         97         99         102         104         106         109         113         117         120         707         101         148         148         151         154         157         160         183         166         169         172         701         148         148         151         154         157         160         183         166         169         172	in Feet	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
694		0	0	0	0	0	0	0	0	0	0
695         12         13         15         16         17         19         20         22         24         25           696         27         29         31         33         35         37         39         41         43         45           697         47         49         52         54         56         58         61         63         68         68           698         71         73         75         78         80         82         85         87         90         92           699         94         97         99         102         104         106         109         113         117         120           700         123         126         128         131         133         135         137         139         142         144           701         146         148         151         154         157         160         163         166         169         172           702         175         178         181         184         187         190         193         196         199         202           703         205         207         211		1	1	1	2	2	2	2	3	3	3
686         27         29         31         33         35         37         39         41         43         45           697         47         49         52         54         56         58         61         63         66         68           698         71         73         75         78         80         82         85         87         90         92           699         94         97         99         102         104         106         109         113         117         120           700         123         126         128         131         133         135         137         139         142         144           701         146         148         151         154         157         160         163         166         169         172           702         175         178         181         184         187         190         193         196         199         202           703         205         207         211         214         217         221         224         227         230         233           704         237         240		4	4	5	5	6	7	8	9	10	11
697         47         49         52         54         56         58         61         63         66         68           698         71         73         75         78         80         82         85         87         90         92           699         94         97         99         102         104         106         109         113         117         120           700         123         126         128         131         133         135         137         139         142         144           701         146         148         151         154         157         180         163         166         169         172           702         175         178         181         184         187         190         193         196         199         202           704         237         240         243         245         248         251         254         257         259         262           705         265         268         271         274         278         281         285         288         292         295           706         298         301<			13	15	16	17	19	20	22	24	25
698         71         73         75         78         80         82         85         87         90         92           699         94         97         99         102         104         106         109         113         117         120           700         123         126         128         131         133         135         137         139         142         144           701         146         148         151         154         157         160         163         166         169         172           702         175         178         181         184         187         190         193         196         199         202           703         205         207         211         214         217         221         224         227         230         233           704         237         240         243         245         248         251         254         257         259         262           706         288         301         304         307         310         312         315         318         321         324           707         327		27	29	31	33	35	37	39	41	43	45
699         94         97         99         102         104         106         109         113         117         120           700         123         126         128         131         133         135         137         139         142         144           701         146         148         151         154         157         160         163         166         169         172           702         175         178         181         184         187         190         193         196         199         202           703         205         207         211         214         217         221         224         227         230         233           704         237         240         243         245         248         251         254         257         259         262           705         265         268         271         274         278         281         285         288         292         295           706         298         301         304         307         310         312         315         318         321         384         350           707 </td <td></td> <td>47</td> <td>49</td> <td>52</td> <td>54</td> <td>56</td> <td>58</td> <td>61</td> <td>63</td> <td>66</td> <td>68</td>		47	49	52	54	56	58	61	63	66	68
700         123         126         128         131         133         135         137         139         142         144           701         146         148         151         154         157         160         163         166         169         172           702         175         178         181         184         187         190         193         196         199         202           703         205         207         211         214         217         221         224         227         230         233           704         237         240         243         245         248         251         285         288         292         295           705         265         268         271         274         278         281         285         288         292         295           706         298         301         304         307         310         312         315         318         321         324           707         327         330         332         335         338         341         343         368         370         373         376           70		71	73	75	78	80	82	85	87	90	92
701         146         148         151         154         157         160         163         166         169         172           702         175         178         181         184         187         190         193         196         199         202           703         205         207         211         214         217         221         224         227         230         233           704         237         240         243         245         248         251         254         257         259         262           705         265         268         271         274         278         281         285         288         292         295           706         298         301         304         307         310         312         315         318         321         324           707         327         330         332         335         338         341         343         346         348         350           708         353         355         357         360         362         365         368         370         373         373         376           70		94	97	99	102	104	106	109	113	117	120
702         175         178         181         184         187         190         193         196         199         202           703         205         207         211         214         217         221         224         227         230         233           704         237         240         243         245         248         251         254         257         259         262           705         265         268         271         274         278         281         285         288         292         295           706         298         301         304         307         310         312         315         318         321         324           707         327         330         332         335         338         341         343         346         348         350           708         353         355         357         360         362         365         368         370         373         376           709         379         382         385         388         391         393         396         399         402         405           710         40	700	123	126	128	131	133	135	137	139	142	144
703         205         207         211         214         217         221         224         227         230         233           704         237         240         243         245         248         251         254         257         259         262           705         265         268         271         274         278         281         285         288         292         295           706         298         301         304         307         310         312         315         318         321         324           707         327         330         332         335         338         341         343         346         348         350           708         353         355         357         360         362         365         368         370         373         376           709         379         382         385         388         391         393         396         399         402         405           710         408         411         414         418         421         424         428         432         436         440           711         44		146	148	151	154		160	163	166	169	172
704         237         240         243         245         248         251         254         257         259         262           705         265         268         271         274         278         281         285         288         292         295           706         298         301         304         307         310         312         315         318         321         324           707         327         330         332         335         338         341         343         346         348         350           708         353         355         367         360         362         365         368         370         373         376           709         379         382         385         388         391         393         396         399         402         405           710         408         411         414         418         421         424         428         432         436         440           711         444         448         452         456         460         464         468         472         476         480           712         48		175	178	181	184	187	190	193	196	199	202
705         265         268         271         274         278         281         285         288         292         295           706         298         301         304         307         310         312         315         318         321         324           707         327         330         332         335         338         341         343         346         348         350           709         379         382         385         388         391         393         396         399         402         405           710         408         411         414         418         421         424         428         432         436         440           711         444         448         452         456         460         464         468         472         476         480           711         484         488         492         495         499         503         507         511         515         519           713         523         528         532         537         541         546         551         556         561         567           714         57		205	207	211	214	217	221	224	227	230	233
706         298         301         304         307         310         312         315         318         321         324           707         327         330         332         335         338         341         343         346         348         355           708         353         355         357         360         362         365         368         370         373         376           709         379         382         385         388         391         393         396         399         402         405           710         408         411         414         418         421         424         428         432         436         440           711         444         448         452         456         460         464         468         472         476         480           711         444         448         489         495         499         503         507         511         515         519           713         523         528         532         537         541         546         551         556         561         667           714         57		237	240	243	245	248	251	254	257	259	262
707         327         330         332         335         338         341         343         346         348         350           708         353         355         357         360         362         365         368         370         373         376           709         379         382         385         388         391         393         396         399         402         405           710         408         411         414         418         421         424         428         432         436         440           711         444         448         452         456         460         464         468         472         476         480           712         484         488         492         495         499         503         507         511         515         519           713         523         528         532         537         541         546         651         567         711         572         578         584         590         595         600         606         611         616         620           715         625         630         635		265	268	271	274	278	281	285	288	292	295
708         353         355         357         360         362         365         368         370         373         376           709         379         382         385         388         391         393         396         399         402         405           710         408         411         414         418         424         428         432         436         440           711         444         448         452         456         460         464         468         472         476         480           712         484         488         492         495         499         503         507         511         515         519           713         523         528         532         537         541         546         551         556         561         567           714         572         578         584         590         595         600         606         611         616         620           715         625         630         635         639         644         649         654         659         664         668           716         673         67		298	301	304	307	310	312	315	318	321	324
709         379         382         385         388         391         393         396         399         402         405           710         408         411         414         418         421         424         428         432         436         440           711         444         448         452         456         460         464         468         472         476         480           712         484         488         492         495         499         503         507         511         515         519           713         523         528         532         537         541         546         551         556         561         567           714         572         578         584         590         595         600         606         611         616         620           715         625         630         635         639         644         649         654         659         664         668           716         673         677         682         686         691         695         700         704         708         713           717         71		327	330	332	335	338	341	343	346	348	350
710         408         411         414         418         421         424         428         432         436         440           711         444         448         452         456         460         464         468         472         476         480           712         484         488         492         495         499         503         507         511         515         519           713         523         528         532         537         541         546         551         556         561         567           714         572         578         584         590         595         600         606         611         616         620           715         625         630         635         639         644         649         654         659         664         668           716         673         677         682         686         691         695         700         704         708         713           717         718         723         728         734         739         745         750         756         762         767           718         77		353	355	357	360	362	365	368	370	373	376
711         444         448         452         456         460         464         468         472         476         480           712         484         488         492         495         499         503         507         511         515         519           713         523         528         532         537         541         546         551         556         561         567           714         572         578         584         590         595         600         606         611         616         620           715         625         630         635         639         644         649         654         659         664         668           716         673         677         682         686         691         695         700         704         708         713           717         718         772         777         781         786         790         795         799         804         808         812           719         817         821         825         830         834         839         843         848         853         857           72				385	388		393	396	399	402	405
712         484         488         492         495         499         503         507         511         515         519           713         523         528         532         537         541         546         551         556         561         567           714         572         578         584         590         595         600         606         611         616         620           715         625         630         635         639         644         649         654         659         664         668           716         673         677         682         686         691         695         700         704         708         713           717         718         723         728         734         739         745         750         756         762         767           718         772         777         781         786         790         795         799         804         808         812           719         817         821         825         830         834         839         843         848         853         857           720         86		408	411	414	418	421	424	428	432	436	440
713         523         528         532         537         541         546         551         556         561         567           714         572         578         584         590         595         600         606         611         616         620           715         625         630         635         639         644         649         654         659         664         668           716         673         677         682         686         691         695         700         704         708         713           717         718         723         728         734         739         745         750         756         762         767           718         772         777         781         786         790         795         799         804         808         812           719         817         821         825         830         834         839         843         848         853         857           720         862         866         871         876         881         886         891         896         901         906           721         96		444	448		456		464	468	472	476	480
714         572         578         584         590         595         600         606         611         616         620           715         625         630         635         639         644         649         654         659         664         668           716         673         677         682         686         691         695         700         704         708         713           717         718         723         728         734         739         745         750         756         762         767           718         772         777         781         786         790         795         799         804         808         812           719         817         821         825         830         834         839         843         848         853         857           720         862         866         871         876         881         886         891         896         901         906           721         911         916         921         926         931         937         943         949         956         963           722         96		484	488	492	495	499	503	507	511	515	519
715         625         630         635         639         644         649         654         659         664         668           716         673         677         682         686         691         695         700         704         708         713           717         718         723         728         734         739         745         750         756         762         767           718         772         777         781         786         790         795         799         804         808         812           719         817         821         825         830         834         839         843         848         853         857           720         862         866         871         876         881         886         891         896         901         906           721         911         916         921         926         931         937         943         949         956         963           722         969         975         981         987         992         997         1,002         1,007         1,012         1,016           723		523	528	532	537	541	546	551	556	561	567
716         673         677         682         686         691         695         700         704         708         713           717         718         723         728         734         739         745         750         756         762         767           718         772         777         781         786         790         795         799         804         808         812           719         817         821         825         830         834         839         843         848         853         857           720         862         866         871         876         881         886         891         896         901         906           721         911         916         921         926         931         937         943         949         956         963           722         969         975         981         987         992         997         1,002         1,007         1,012         1,016           723         1,022         1,027         1,032         1,038         1,043         1,049         1,054         1,060         1,066         1,072		572					600		611		620
717         718         723         728         734         739         745         750         756         762         767           718         772         777         781         786         790         795         799         804         808         812           719         817         821         825         830         834         839         843         848         853         857           720         862         866         871         876         881         886         891         896         901         906           721         911         916         921         926         931         937         943         949         956         963           722         969         975         981         987         992         997         1,002         1,007         1,012         1,016           723         1,022         1,027         1,032         1,038         1,043         1,049         1,054         1,060         1,066         1,072           724         1,078         1,085         1,092         1,098         1,104         1,110         1,116         1,121         1,128         1,134 </td <td></td> <td>625</td> <td></td> <td>635</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>668</td>		625		635							668
718         772         777         781         786         790         795         799         804         808         812           719         817         821         825         830         834         839         843         848         853         857           720         862         866         871         876         881         886         891         896         901         906           721         911         916         921         926         931         937         943         949         956         963           722         969         975         981         987         992         997         1,002         1,007         1,012         1,016           723         1,022         1,027         1,032         1,038         1,043         1,049         1,054         1,060         1,066         1,072           724         1,078         1,085         1,092         1,098         1,104         1,110         1,116         1,121         1,128         1,134           725         1,140         1,146         1,152         1,158         1,164         1,169         1,175         1,181         1,187         <		673	677	682	686	691	695	700	704	708	713
719         817         821         825         830         834         839         843         848         853         857           720         862         866         871         876         881         886         891         896         901         906           721         911         916         921         926         931         937         943         949         956         963           722         969         975         981         987         992         997         1,002         1,007         1,012         1,016           723         1,022         1,027         1,032         1,038         1,043         1,049         1,054         1,060         1,066         1,072           724         1,078         1,085         1,092         1,098         1,104         1,110         1,116         1,121         1,128         1,134           725         1,140         1,146         1,152         1,158         1,164         1,169         1,175         1,181         1,187         1,193           726         1,199         1,205         1,212         1,219         1,226         1,233         1,239         1,246		718	723	728					756		767
720         862         866         871         876         881         886         891         896         901         906           721         911         916         921         926         931         937         943         949         956         963           722         969         975         981         987         992         997         1,002         1,007         1,012         1,016           723         1,022         1,027         1,032         1,038         1,043         1,049         1,054         1,060         1,066         1,072           724         1,078         1,085         1,092         1,098         1,104         1,110         1,116         1,121         1,128         1,134           725         1,140         1,146         1,152         1,158         1,164         1,169         1,175         1,181         1,187         1,193           726         1,199         1,205         1,212         1,219         1,226         1,233         1,239         1,246         1,253         1,260           727         1,267         1,274         1,281         1,288         1,296         1,303         1,311         1,3								799	804	808	
721         911         916         921         926         931         937         943         949         956         963           722         969         975         981         987         992         997         1,002         1,007         1,012         1,016           723         1,022         1,027         1,032         1,038         1,043         1,049         1,054         1,060         1,066         1,072           724         1,078         1,085         1,092         1,098         1,104         1,110         1,116         1,121         1,128         1,134           725         1,140         1,146         1,152         1,158         1,164         1,169         1,175         1,181         1,187         1,193           726         1,199         1,205         1,212         1,219         1,226         1,233         1,239         1,246         1,253         1,260           727         1,267         1,274         1,281         1,288         1,296         1,303         1,311         1,318         1,324         1,331           728         1,338         1,345         1,351         1,358         1,364         1,371         1,377<		817			830	834	839		848	853	857
722         969         975         981         987         992         997         1,002         1,007         1,012         1,016           723         1,022         1,027         1,032         1,038         1,043         1,049         1,054         1,060         1,066         1,072           724         1,078         1,085         1,092         1,098         1,104         1,110         1,116         1,121         1,128         1,134           725         1,140         1,146         1,152         1,158         1,164         1,169         1,175         1,181         1,187         1,193           726         1,199         1,205         1,212         1,219         1,226         1,233         1,239         1,246         1,253         1,260           727         1,267         1,274         1,281         1,288         1,296         1,303         1,311         1,318         1,324         1,331           728         1,338         1,345         1,351         1,358         1,364         1,371         1,377         1,384         1,390         1,396           729         1,403         1,410         1,417         1,424         1,429         1,434 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>896</td> <td></td> <td></td>									896		
723         1,022         1,027         1,032         1,038         1,043         1,049         1,054         1,060         1,066         1,072           724         1,078         1,085         1,092         1,098         1,104         1,110         1,116         1,121         1,128         1,134           725         1,140         1,146         1,152         1,158         1,164         1,169         1,175         1,181         1,187         1,193           726         1,199         1,205         1,212         1,219         1,226         1,233         1,239         1,246         1,253         1,260           727         1,267         1,274         1,281         1,288         1,296         1,303         1,311         1,318         1,324         1,331           728         1,338         1,345         1,351         1,358         1,364         1,371         1,377         1,384         1,390         1,396           729         1,403         1,410         1,417         1,424         1,429         1,434         1,438         1,442         1,446         1,450           730         1,454         1,458         1,462         1,473         1,476         <								943	949		
724         1,078         1,085         1,092         1,098         1,104         1,110         1,116         1,121         1,128         1,134           725         1,140         1,146         1,152         1,158         1,164         1,169         1,175         1,181         1,187         1,193           726         1,199         1,205         1,212         1,219         1,226         1,233         1,239         1,246         1,253         1,260           727         1,267         1,274         1,281         1,288         1,296         1,303         1,311         1,318         1,324         1,331           728         1,338         1,345         1,351         1,358         1,364         1,371         1,377         1,384         1,390         1,396           729         1,403         1,410         1,417         1,424         1,429         1,434         1,438         1,442         1,446         1,450           730         1,454         1,458         1,462         1,473         1,476         1,479         1,481         1,484         1,487         1,490           731         1,492         1,495         1,498         1,501         1,503         <						992		1,002	1,007	1,012	1,016
725         1,140         1,146         1,152         1,158         1,164         1,169         1,175         1,181         1,187         1,193           726         1,199         1,205         1,212         1,219         1,226         1,233         1,239         1,246         1,253         1,260           727         1,267         1,274         1,281         1,288         1,296         1,303         1,311         1,318         1,324         1,331           728         1,338         1,345         1,351         1,358         1,364         1,371         1,377         1,384         1,390         1,396           729         1,403         1,410         1,417         1,424         1,429         1,434         1,438         1,442         1,446         1,450           730         1,454         1,458         1,462         1,473         1,476         1,479         1,481         1,484         1,487         1,490           731         1,492         1,495         1,498         1,501         1,503         1,506         1,509         1,512         1,515         1,518           732         1,520         1,523         1,526         1,529         1,532         <		1,022	1,027	1,032	1,038	1,043	1,049	1,054	1,060	1,066	1,072
726         1,199         1,205         1,212         1,219         1,226         1,233         1,239         1,246         1,253         1,260           727         1,267         1,274         1,281         1,288         1,296         1,303         1,311         1,318         1,324         1,331           728         1,338         1,345         1,351         1,358         1,364         1,371         1,377         1,384         1,390         1,396           729         1,403         1,410         1,417         1,424         1,429         1,434         1,438         1,442         1,446         1,450           730         1,454         1,458         1,462         1,473         1,476         1,479         1,481         1,484         1,487         1,490           731         1,492         1,495         1,498         1,501         1,503         1,506         1,509         1,512         1,515         1,518           732         1,520         1,523         1,526         1,529         1,532         1,535         1,538         1,541         1,544         1,547		1,078	1,085	1,092	1,098	1,104	1,110	1,116	1,121	1,128	1,134
727         1,267         1,274         1,281         1,288         1,296         1,303         1,311         1,318         1,324         1,331           728         1,338         1,345         1,351         1,358         1,364         1,371         1,377         1,384         1,390         1,396           729         1,403         1,410         1,417         1,424         1,429         1,434         1,438         1,442         1,446         1,450           730         1,454         1,458         1,462         1,473         1,476         1,479         1,481         1,484         1,487         1,490           731         1,492         1,495         1,498         1,501         1,503         1,506         1,509         1,512         1,515         1,518           732         1,520         1,523         1,526         1,529         1,532         1,535         1,538         1,541         1,544         1,547											
728         1,338         1,345         1,351         1,358         1,364         1,371         1,377         1,384         1,390         1,396           729         1,403         1,410         1,417         1,424         1,429         1,434         1,438         1,442         1,446         1,450           730         1,454         1,458         1,462         1,473         1,476         1,479         1,481         1,484         1,487         1,490           731         1,492         1,495         1,498         1,501         1,503         1,506         1,509         1,512         1,515         1,518           732         1,520         1,523         1,526         1,529         1,532         1,535         1,538         1,541         1,544         1,547			1,205								
729     1,403     1,410     1,417     1,424     1,429     1,434     1,438     1,442     1,446     1,450       730     1,454     1,458     1,462     1,473     1,476     1,479     1,481     1,484     1,487     1,490       731     1,492     1,495     1,498     1,501     1,503     1,506     1,509     1,512     1,515     1,518       732     1,520     1,523     1,526     1,529     1,532     1,535     1,538     1,541     1,544     1,547		1,267	1,274	1,281	1,288	1,296	1,303	1,311	1,318	1,324	1,331
730     1,454     1,458     1,462     1,473     1,476     1,479     1,481     1,484     1,487     1,490       731     1,492     1,495     1,498     1,501     1,503     1,506     1,509     1,512     1,515     1,518       732     1,520     1,523     1,526     1,529     1,532     1,535     1,538     1,541     1,544     1,547		1,338	1,345	1,351	1,358	1,364	1,371	1,377	1,384	1,390	1,396
731         1,492         1,495         1,498         1,501         1,503         1,506         1,509         1,512         1,515         1,518           732         1,520         1,523         1,526         1,529         1,532         1,535         1,538         1,541         1,544         1,547		1,403	1,410	1,417				1,438		1,446	1,450
732 1,520 1,523 1,526 1,529 1,532 1,535 1,538 1,541 1,544 1,547		1,454	1,458	1,462	1,473	1,476	1,479	1,481	1,484	1,487	1,490
		1,492	1,495	1,498	1,501	1,503	1,506	1,509	1,512	1,515	1,518
733 1,550 1,553 1,556 1,559 1,563 1,568			1,523	1,526	1,529	1,532	1,535	1,538	1,541	1,544	1,547
	733	1,550	1,553	1,556	1,559	1,563	1,568				



Appendix C: Area and Capacity Curves

